Exchange rates and commodity prices: measuring causality at multiple horizons [*]

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ABSTRACT

Different causal mechanisms have been proposed to link commodity prices and exchange rates, with opposing implications. We examine these causal relationships empirically, using data on three commodities (crude oil, gold, copper) and four countries (Canada, Australia, Norway, Chile), over the period 1986-2015. To go beyond pure significance tests of Granger non-causality and provide a relatively complete picture of the links, measures of the strength of causality for different horizons and directions are estimated and compared. Since low-frequency data may easily fail to capture important features of the relevant causal links, daily and some 5-minute data are exploited. Both unconditional and conditional (given general stock market conditions and short-term interest rates) causality measures are considered, and allowance for "dollar effects" is made by considering non-U.S. dollar exchange rates. We identify clear causal patterns: (1) there is evidence of Granger-causality between commodity prices and exchange rates in both directions across multiple horizons, but the statistical evidence and measured intensity of the effects are much stronger in the direction of commodity prices to exchange rates, especially at horizon one: the ratios of causality measures in two different directions can be quite high; (2) causality is stronger at short horizons, and becomes weaker as the horizon increases; (3) conditioning on equity prices (the S&P500) does not change the patterns of causality measures found in the unconditional cases; (4) the main results are robust to eliminating U.S.-dollar denomination effects and including a short-term interest rate as the conditioning variable. In contrast with earlier results on the non-predictability of exchange rates, we find that the macroeconomic/trade-based mechanism plays a central role in exchange-rate dynamics, despite the financial feature of these markets.

Key words: multi-horizon causality; causality measures; commodity prices; exchange rates; spurious causality.

Journal of Economic Literature classification: F31, G15, G17.

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1. Introduction

The dynamic relationship between commodity prices and exchange rates has attracted much attention from both researchers and practitioners. Two main explanations have been proposed. The first one suggests that changes in a commodity price lead to changes in the exchange rate of the corresponding commodity currency. This idea commonly appears in both the research literature [see, for example, Chen and Rogoff (2003) and Chen (2004)] and press commentaries.[1] The second explanation stresses the financial and speculative features of foreign exchange markets: exchange rates can help predict economic fundamentals including commodity prices; see, for example, Meese and Rogoff (1983), Engel and West (2005), Cheung, Chinn and Pascual (2005), Rogoff and Stavrakeva (2008), Chen, Rogoff and Rossi (2010), and Rossi (2013). Following the first mechanism, commodity prices should help predict exchange-rate movements. According to the second one, the reverse should happen. Thus, a central difference between these two alternative explanations lies in the direction of causality in the sense of Wiener-Granger.[2]

The first theory relies on macroeconomic and trade-theory arguments. For a small open economy whose exports depend heavily on a particular commodity (for example, gold for Australia, crude oil for Canada and Norway, copper for Chile), an increase in the price of that commodity should produce an upward pressure on the demand for its currency, which leads to an appreciation. For instance, while crude oil is the largest Canadian export, Canada's total crude oil production is a small share of world output. The price of oil is determined by global supply and demand conditions to which Canada contributes only modestly, while a change in the price of oil has a large effect on the value of Canadian exports. This mechanism can be justified in sticky-price open economy models with non-traded goods, a portfolio-balance model, and the terms-of-trade hypothesis; see Chen and Rogoff (2003) and Chen (2004). This type of explanation suggests that exchange-rate movements can be predicted by economic variables. However, statistical evidence shows that it is generally difficult to forecast exchange rates, so economic models of exchange-rate determination do not fare well from the empirical viewpoint.[3]

Instead, according to the second theory, exchange rates are determined – like most asset prices – by the net present value of fundamentals (including commodity prices), which implies that exchange rates should lead and therefore Granger-cause commodity prices; see Obstfeld and Rogoff (1996), Engel and West (2005), Chen et al. (2010) and Alquist, Kilian and Vigfusson (2012).[4]

[1] For example, David Parkinson writes in the Globe and Mail (Report on Business, 10 April 2010, B14): "When analyzing the loonie, always look at oil"; "loonie" is a colloquialism for the Canadian dollar, a reference to the image of a loon on the coin. In Bloomberg Businessweek (April 18, 2013), Sebastian Boyd states: "Chilean Peso declines as principal export copper reaches new low". In the Wall Street Journal (July 5, 2013), Vincent Cignarella writes: " … a rise in the price of the precious metal would do wonders to boost the fortunes of the Australian dollar".

[2] This is the concept of causality that will be used throughout.

[3] For more general discussions of the theory and empirical evidence on exchange rate markets, see e.g. Levich (1985), Baillie and McMahon (1989), Baillie and Bollerslev (1990), Baillie and Bollerslev (1994), Frankel and Rose (1995), Froot and Rogoff (1995), Isard (1995), Obstfeld and Rogoff (1996), Mark (2001), Sarno and Taylor (2002) and Kilian and Taylor (2003).

[4] For work on forecasting commodity prices (especially energy prices), see also Schwartz (1997), Schwartz and Smith (2000), Pindyck (2001), Hamilton (2009), Alquist and Kilian (2010), Reeve and Vigfusson (2011), Bernard, Dufour, Khalaf and Kichian (2012), Baumeister and Kilian (2012), Baumeister and Kilian (2013), Baumeister, Kilian and Zhou

Here we examine empirically the causal relationship between commodity prices and nominal exchange rates, using data on three commodities (crude oil, gold, copper) and four countries (Canada, Australia, Norway, Chile), over the period 1986-2015. We emphasize five issues which should be taken into account.

First, predictability and dynamic responses may depend on the time horizon, so it is important to assess these links across different horizons. In particular, looking at multiple-horizon causality does allow one to account for indirect causal links – which go through different variables across time – and may help to eliminate spurious findings of causation; see Dufour and Renault (1998).

Second, given that causal links may theoretically exist in all directions, it is of interest to determine which links – in terms of direction and time horizon – matter most. This can be done using measures of the strength of causal links. Significance tests (for non-causality) are inappropriate for that purpose: a large effect (from an economic viewpoint) may not be statistically significant because the data do not allow one to measure it precisely, and an economically negligible effect may be statistically significant because the effect, while small, can be very precisely estimated. It is much more informative to parameterize the relevant effects, compute point estimates for these parameters, and eventually confidence sets; see Dufour and Taamouti (2010) and Dufour, Garcia and Taamouti (2012). Non-causality tests can provide evidence on the presence forecast improvements from the inclusion of different past variables, but do not indicate the magnitudes of such improvements.

Third, the proposed measures should be intuitive and easy to interpret without a highly restrictive parametric model. In particular, they should allow for a wide spectrum of dynamic structures. To this end, we use here the methodology of Dufour and Taamouti (2010) and Dufour et al. (2012).

Fourth, it is well known that Granger causality is not generally invariant to aggregation: high-frequency data may reveal patterns which are aggregated away in low-frequency data, and causality in low-frequency data can also be spurious; see Tiao and Wei (1976), Wei (1982, 1990), Marcellino (1999), Breitung and Swanson (2002), and Silvestrini and Veredas (2008). Indeed, as stressed in Dufour and Renault (1998), the interpretation of Granger causality depends on the forecast horizon and data frequency. Data on commodity prices and exchange rates are originally generated at very high frequency. Quarterly data typically used in macroeconomic studies are obtained by aggregating high-frequency data over time. Spurious causality can be induced when intervals between microeconomic decisions of economic agents are finer than those between sample observations.

Fifth, commodity prices and exchange rates are set in active financial markets. Movements in such markets can be fast or short-lived, so low-frequency data may easily fail to capture causal links.

No earlier study of the behavior of exchange rates meets these objectives. The closest papers include studies of the relationship between real exchange rates and real commodity prices based on low-frequency (*e.g.*, quarterly) data; see Gruen and Wilkinson (1994), Amano and van Norden (1995, 1998*a*), Amano and van Norden (1998*b*), Chen and Rogoff (2003), Cashina, Céspedes and Sahay (2004), Issa, Lafrance and Murray (2008). Significance tests of the predictive relationship between nominal exchange rates and commodity prices (including tests of conventional Granger non-causality) are also reported by Chen (2004), Chen et al. (2010), Alquist et al. (2012), and Ferraro, Rogoff and Rossi (2012). None of above studies considers the magnitude of prediction improvements using measures of the strength of causality.

(2013) and the references therein.

In this paper, we assess the strength of the underlying linkages between commodity prices and exchange rates by estimating causality measures at several horizons in both directions. The measures used are based on the concepts and statistical methodology – including both point estimates (of causality measures) and confidence intervals – described in Dufour and Taamouti (2010) for a general time-series framework, and Dufour et al. (2012) in the context of high-frequency data (as in this paper). In particular, the statistical setup we consider allows for general assumptions, such as stationary invertible vector autoregressive moving average (VARMA) models. Both unconditional and conditional (given stock price and interest rate movements) measures are considered.

We examine these causal relationships empirically, using data on three commodities (crude oil, gold, copper) and four countries (Canada, Australia, Norway, Chile), over the period 1986-2015. In conditional causality analyses, we include an indicator of the level of equity prices (the S&P500 index) or a short-term interest rate. To account for possible spurious comovements due to the fact that exchange rates and commodity prices are all denominated in U.S. dollars ("dollar effects"), we also consider some alternative currency benchmarks.

Section 2 introduces the framework we use, involving the statistical concepts of multi-horizon causality and measures. Section 3 gives a detailed description of data used in this study and reports and discusses the empirical results. Section 4 concludes.

2. Framework

The main objective of this paper is to examine high-frequency causality between commodity prices and exchange rates using daily and intra-day data. In this section, we introduce the statistical concepts of multi-horizon causality and causality measures that we use.

2.1. Causality at different horizons

Granger (1969) introduced the concept of causality in terms of predictability at horizon one of a (vector) variable X from its own past, the past of another (vector) variable Y, and possibly a vector Z of auxiliary variables; this has come to be known as Granger causality. This concept has become a fundamental notion for studying dynamic relationships among time series. An important extension was proposed by Dufour and Renault (1998) who generalized the notion of Granger causality by considering linear causality at a given (arbitrary) horizon h and derived necessary and sufficient conditions for non-causality between variables up to any given horizon h ($1 \leq h \leq \infty$), allowing the possibility of indirect causality. This indirect causality in the presence of auxiliary variables can be used to distinguish short-run and long-run (non)causality: for example, although Y does not Granger-cause X at horizon one, it may nonetheless help to predict X several periods ahead though transmission by a vector Z of auxiliary variables. The importance of the distinction between correlation and causality is also underscored when considering horizons longer than one period.

Dufour and Renault (1998) defined linear causality at any given horizon $h \geq 1$ in terms of orthogonality between subspaces of a Hilbert space of random variables with finite second moments. We will adopt the notation used in Dufour and Taamouti (2010). We denote by L^2 a Hilbert space of real random variables with finite second moments. Define the "reference information set" $I =$

$\{I(t) : t \in \mathbb{Z},\ t > \omega\}$ and $t < t' \Rightarrow I(t) \subseteq I(t')$ for all $t > \omega$, where $I(t)$ is defined on Hilbert subspace of L^2, $\omega \in \mathbb{Z} \cup \{-\infty\}$ represents a "starting point", and $\mathbb{Z}$ is the set of the integers. Let H be a (possibly empty) Hilbert subspace of L^2, which contains information common to all $I(t)$ [*e.g.*, the constant in a regression model], and assume $H \subseteq I(t)$. Consider three multivariate stochastic processes: $X = \{X(t) : t \in \mathbb{Z},\ t > \omega\}$, $Y = \{Y(t) : t \in \mathbb{Z},\ t > \omega\}$ and $Z = \{Z(t) : t \in \mathbb{Z}, t > \omega\}$, where $X(t) = \big(x_1(t), \ldots, x_{m_1}(t)\big)'$, $Y(t) = \big(y_1(t), \ldots, y_{m_2}(t)\big)'$, $Z(t) = \big(z_1(t), \ldots, z_{m_3}(t)\big)'$, with numbers of components $m_1 \geq 1$, $m_2 \geq 1$, $m_3 \geq 0$, and $x_i(t)$, $y_i(t)$, $z_i(t) \in L^2$, for all i. Denote by $X(\omega,t]$, $Y(\omega,t]$ and $Z(\omega,t]$ the Hilbert spaces spanned by the components of variables X, Y and Z respectively up to time t. Then information sets $I_X(t)$ and $I_{XY}(t)$ are defined as $I_X(t) = I(t) + X(\omega,t]$ and $I_{XY}(t) = I(t) + X(\omega,t] + Y(\omega,t]$, and $Z(\omega,t]$ is assumed to be included in $I(t)$.

For any information set $B(t)$ (some Hilbert subspace of L^2), given a positive integer h, we denote by $P[X(t+h) \mid B(t)]$ the best linear forecast of $X(t+h)$ based on the information set $B(t), by$

$$U_L[X(t+h) \mid B(t)] = X(t+h) - P[X(t+h) \mid B(t)]$$

the corresponding linear forecast error, and by

$$\Sigma[X(t+h) \mid B(t)] = \mathsf{E}\left\{U_L[X(t+h) \mid B(t)]\, U_L'[X(t+h) \mid B(t)]\right\}$$

the variance-covariance matrix of the linear forecast error (or mean squared error, MSE). Thus we have the following definition of non-causality at any given horizon $h \geq 1$ [see Dufour and Renault (1998) and Dufour and Taamouti (2010)].

Definition 2.1 NON-CAUSALITY AT HORIZON *h*. *Y does not cause X at horizon h given I, denoted* $Y \underset{h}{\nrightarrow} X \mid I$, *iff*

$$P[X(t+h) \mid I_X(t)] = P[X(t+h) \mid I_{XY}(t)].$$

We can define non-causality from X to Y at horizon h similarly. This definition concerns the conditional non-causality with auxiliary variables, which may transmit indirect causality between variables at horizons higher than one, even if there is no direct causality at horizon one. If Z is dropped from the information set ($m_3 = 0$), then the above definition represents unconditional non-causality. In the absence of auxiliary variables, unconditional non-causality at horizon one implies non-causality at any horizon h (which can be unbounded); see Dufour and Renault (1998).

2.2. Measuring causality across horizons

Rejecting non-causality hypotheses in statistical tests implies that certain variables can help in forecasting others [Dufour, Pelletier and Renault (2006)]. Of course, statistical significance depends on the data and test power, and the outcomes of such tests do not represent the magnitude of causality. Geweke (1982, 1984) interpreted causality measures as the proportional reduction in the forecast error variance of a variable available by taking into account the past of other variables. Dufour and Taamouti (2010) make multi-horizon extensions of such measures in the context of a set of linear invertible processes (including VAR, VMA, and VARMA). The latter authors note that "building causality measures at different horizons, along with associated confidence intervals, can yield a much more informative analysis of Granger causality than tests of non-causality."

Following Dufour and Taamouti (2010), we measure causality at horizon $h \geq 1$ as follows.

Definition 2.2 CAUSALITY MEASURE AT HORIZON *h*. *For* $h \geq 1$,

$$C_L\big(Y \underset{h}{\rightarrow} X \mid I\big) = \ln\left[\frac{\det\{\Sigma[X(t+h) \mid I_X(t)]\}}{\det\{\Sigma[X(t+h) \mid I_{XY}(t)]\}}\right] \tag{2.1}$$

is the mean-square causality measure from Y to X at horizon h, given I.

A causality measure from X to Y at horizon h is defined in a similar way. For $m_1 = m_2 = 1$, the above definition reduces to

$$C_L\big(Y \underset{h}{\rightarrow} X \mid I\big) = \ln\left[\frac{\sigma^2[X(t+h) \mid I_X(t)]}{\sigma^2[X(t+h) \mid I_{XY}(t)]}\right].$$

This definition allows for conditional causality with auxiliary variables. If Z is empty $(m_3 = 0)$, Definition 2.2 defines an unconditional causality measure. This causality measure is nonnegative, and zero if and only if there is no causality at the horizon considered; the higher the value of the measure, the stronger is the causal relationship. When non-causality tests reject in both directions, causality may nonetheless be much stronger in one direction, the feature revealed by causality measures. Furthermore, confidence intervals for causality measures can provide more powerful tests for non-causality at any given horizon, and help determine how long the causal effects will last.

2.3. Causality measures in VARMA models

We now describe parametric representations of causality measures in the context of linear invertible VARMA models of finite order, which will be used in the empirical analyses below. For simplicity, we assume $X(t)$, and $Y(t)$ are univariate processes ($m_1 = m_2 = 1$). The discrete $m \times 1$ vector process with zero mean $W(t) = \big(X(t)', Y(t)', Z(t)'\big)'$ defined on L^2 is characterized by a stationary and invertible VARMA(p,q) model,

$$W(t) = \sum_{i=1}^{p} \phi_i W(t-i) + \sum_{j=1}^{q} \varphi_j u(t-j) + u(t) \tag{2.2}$$

where $u(t)$ is m-dimensional white noise process with non-singular variance-covariance matrix Σ_u, and $m = m_1 + m_2 + m_3$. Hereafter, we call $W(t)$ defined in (2.2) the unconstrained model.

To measure causality from Y to X at horizon h, we need to know the structure of the marginal process $W_0(t) = \big(X(t)', Z(t)'\big)'$. According to Lütkepohl (1993), it follows a stationary VARMA($\bar{p} \leq mp,\ \bar{q} \leq (m-1)p+q$):

$$W_0(t) = \sum_{i=1}^{\bar{p}} \bar{\phi}_i W_0(t-i) + \sum_{j=1}^{\bar{q}} \bar{\varphi}_j e(t-j) + e(t) \tag{2.3}$$

where $e(t)$ is m_0-dimensional white noise process with non-singular variance-covariance matrix Σ_e, and $m_0 = m_1 + m_3$. Hereafter, we call $W_0(t)$ defined in (2.3) the constrained model.

Under stationarity, $W(t)$ has a VMA(∞) representation,

$$W(t) = \sum_{j=0}^{\infty} \psi_j u(t-j) \tag{2.4}$$

where $\psi_0 = I_m$, and ψ_j for $j > 0$ can be represented as functions of the ϕ_i and φ_j coefficients. The forecast error of linear forecast of $W(t+h)$ based on $I_W(t)$, and its variance-covariance matrix are given by

$$U_L\left[W(t+h) \mid I_W(t)\right] = \sum_{j=0}^{h-1} \psi_j u(t+h-j) \ , \tag{2.5}$$

$$\Sigma\left[W(t+h) \mid I_W(t)\right] = \sum_{j=0}^{h-1} \psi_j \mathsf{E}\left[u(t+h-j)\, u'(t+h-j)\right] \psi_j' = \sum_{j=0}^{h-1} \psi_j \Sigma_u \psi_j', \tag{2.6}$$

where $I_W(t) = I(t) + X(\omega,t] + Y(\omega,t]$, and the information set $I(t)$ contains $Z(\omega,t]$. Then the unconstrained MSE for the linear forecast of $X(t+h)$ is

$$\sigma^2\left[X(t+h) \mid I_W(t)\right] = \sum_{j=0}^{h-1} J_1 \psi_j \Sigma_u \psi_j' J_1' \tag{2.7}$$

where J_1 is a m-dimensional vector taking the value of one only at the first place, and zero at the other places. Similarly, the constrained model (2.3) can be written as a VMA(∞) model,

$$W_0(t) = \sum_{j=0}^{\infty} \bar{\psi}_j e(t-j), \tag{2.8}$$

where $\bar{\psi}_0 = I_{m_0}$, and $\bar{\psi}_j$ for $j > 0$ are functions of the $\bar{\phi}_i$ and $\bar{\varphi}_j$ coefficients. The forecast error for the linear forecast of $W_0(t+h)$ based on I_{W_0} and its variance-covariance matrix are then given by:

$$U_L\left[W_0(t+h) \mid I_{W_0}(t)\right] = \sum_{j=0}^{h-1} \bar{\psi}_j e(t+h-j) \ , \tag{2.9}$$

$$\Sigma\left[W_0(t+h) \mid I_{W_0}(t)\right] = \sum_{j=0}^{h-1} \bar{\psi}_j \mathsf{E}\left[e(t+h-j)\, e'(t+h-j)\right] \bar{\psi}_j' = \sum_{j=0}^{h-1} \bar{\psi}_j \Sigma_e \bar{\psi}_j' \ . \tag{2.10}$$

where $I_{W_0}(t) = I(t) + X(\omega,t]$ and the information set $I(t)$ contains $Z(\omega,t]$. Thus the constrained MSE for the linear forecast of $X(t+h)$ is

$$\sigma^2\left[X(t+h) \mid I_{W_0}(t)\right] = \sum_{j=0}^{h-1} J_0 \bar{\psi}_j \Sigma_e \bar{\psi}_j' J_0' \tag{2.11}$$

where J_0 is a m_0-dimensional vector taking the value of one only at the first place, and zero at the other places. Consequently, the causality measure from Y to X conditional on I at horizon h can be represented by

$$C_L\big(Y \underset{h}{\rightarrow} X \mid I\big) = \ln\left[\frac{\sigma^2\left[X\left(t+h\right) \mid I_{W_0}\left(t\right)\right]}{\sigma^2\left[X\left(t+h\right) \mid I_W\left(t\right)\right]}\right] = \ln\left[\frac{\sum_{j=0}^{h-1} J_0 \bar{\psi}_j \Sigma_e \bar{\psi}_j' J_0'}{\sum_{j=0}^{h-1} J_1 \psi_j \Sigma_u \psi_j' J_1'}\right]. \tag{2.12}$$

To estimate the causality measure consistently without using maximum likelihood or nonlinear least squares, which involve complicated nonlinear optimization and are therefore difficult to use in the context of bootstrap inference procedures, we use the linear estimation approach proposed in Dufour and Taamouti (2010).

Under the assumption that $W(t)$ is invertible, it can be written as an infinite autoregressive process:

$$W(t) = \sum_{i=1}^{\infty} \pi_i W(t-i) + u(t). \tag{2.13}$$

Given a realization $\{W(1), \ldots, W(T)\}$, we can approximate (2.13) by a finite-order VAR(k) model, where k depends on the sample size T:

$$W(t) = \sum_{i=1}^{k} \pi_{ik} W(t-i) + u_k(t). \tag{2.14}$$

The least-squares estimators of the coefficients $\pi(k) = [\pi_{1k}, \pi_{2k}, \ldots, \pi_{kk}]$ of the VAR(k) model (2.14) and the variance-covariance matrix $\Sigma_{u|k}$ of the error term $u_k(t)$, are denoted as $\hat{\pi}(k)$ and $\hat{\Sigma}_{u|k}$ respectively. Then, we can use $\hat{\pi}(k)$ to calculate the estimator of ψ_j in (2.4), denoted as $\hat{\psi}_{jk}$ for $j = 1, \ldots, h$; see Dufour and Taamouti (2010).

Under general conditions, $W_0(t)$ has a VAR(∞) representation:

$$W_0(t) = \sum_{i=1}^{\infty} \bar{\pi}_i W_0(t-i) + e(t), \tag{2.15}$$

which can also be approximated by a finite-order VAR model, where for the convenience of comparison, we choose the same order k as for the unconstrained model:

$$W_0(t) = \sum_{i=1}^{k} \bar{\pi}_{ik} W_0(t-i) + e_k(t). \tag{2.16}$$

The least-squares estimators of the coefficients $\bar{\pi}(k) = [\bar{\pi}_{1k}, \bar{\pi}_{2k}, \ldots, \bar{\pi}_{kk}]$ of the VAR(k) model (2.16) and the variance-covariance matrix $\Sigma_{e|k}$ of the error term $e_k(t)$ are denoted as $\tilde{\pi}(k)$ and $\tilde{\Sigma}_{e|k}$ respectively. Then, we can use $\tilde{\pi}(k)$ to calculate the estimator of $\bar{\psi}_j$ in (2.8), denoted as $\tilde{\psi}_{jk}$ for $j = 1, \ldots, h$.

Finally, an estimator of the causality measure from Y to X conditional on I at horizon h is given by

$$\hat{C}_L\big(Y \underset{h}{\rightarrow} X \mid I\big) = \ln\left[\frac{\sum_{j=0}^{h-1} J_0 \tilde{\psi}_{jk} \tilde{\Sigma}_{e|k} \tilde{\psi}'_{jk} J'_0}{\sum_{j=0}^{h-1} J_1 \hat{\psi}_{jk} \hat{\Sigma}_{u|k} \hat{\psi}'_{jk} J'_1}\right] \tag{2.17}$$

Dufour and Taamouti (2010) proved the consistency and asymptotic normality of this estimator of the causality measure. That is,

$$T^{1/2}\left[\hat{C}_L\big(Y \underset{h}{\rightarrow} X \mid I\big) - C_L\big(Y \underset{h}{\rightarrow} X \mid I\big)\right] \xrightarrow{d} N\left[0, \sigma_c^2(h)\right]$$

where $\sigma_c^2(h) = D_C \Omega D'_C$, $D_C = \partial C_L\big(Y \underset{h}{\rightarrow} X \mid I\big)/\partial \theta'$, $\theta = \big(\text{vec}(\pi)', \text{vech}(\Sigma_u)'\big)'$, Ω is the asymptotic variance-covariance matrix of $\hat{\theta}$, vec denotes the column stacking operator, and vech is the column stacking operator that stacks the elements on and below the diagonal only. In the empirical implementation below, we estimate the unconditional and conditional causality measures up to horizon ten, based on (2.17), where the value of k is chosen according to the Akaike information criterion (AIC) as suggested by Lewis and Reinsel (1985).

As noted in Dufour and Taamouti (2010), analytical differentiation of the causality measures with respect to θ is very difficult, so a bootstrap approach is a better choice. We therefore use the eight-step residual-based bootstrap method proposed in Dufour and Taamouti (2010) to compute the confidence interval of the causality measure at given horizon h. The asymptotic validity of the residual-based bootstrap $\hat{C}_L^*\big(Y \underset{h}{\rightarrow} X \mid I\big)$ is proven in proposition 8.2 in Dufour and Taamouti (2010):

$$T^{1/2}\big[\hat{C}_L^*\big(Y \underset{h}{\rightarrow} X \mid I\big) - \hat{C}_L\big(Y \underset{h}{\rightarrow} X \mid I\big)\big] \xrightarrow{d} N\left[0, \sigma_c^2(h)\right]$$

where $\sigma_c^2(h)$ is defined as above.

3. Empirical results

In this section, we first describe our data, and then report the results of non-causality tests at horizon one as well as numerical measures of the magnitude of a causal effect at multiple horizons. Because our aim is to identify general patterns rather than to examine a single specific case, we present results on multiple currencies and methods of treating the data. Most of our findings are presented graphically in order to synthesize a large body of evidence in a relatively convenient format. We first report the detailed results, and then summaries of key observations. We then examine the robustness of the main results to alternative choices of exchange-rate numeraire and conditioning variables.

3.1. Data and methods

We consider four commodity-exporting, small open economies with floating exchange rates: Canada (CA), Australia (AU), Norway (NO) and Chile (CL). We use daily data on nominal exchange rates (E), commodity spot prices (P_{com}) and the S&P 500 index price (P_{sp}) and short-term

Table 1: Data description

Canada	
CAD/USD, CAD/GBP, CERI, WTI crude oil price, CTB3	02/01/1986 - 31/07/2015
Australia	
AUD/USD, AUD/JPY, AUD(TWI), Gold price	02/01/1986 - 31/07/2015
Norway	
NOK/USD, Brent crude oil price	20/05/1987 - 31/07/2015
NESR3	08/01/2003 - 31/07/2015
Chile	
CLP/USD, Copper price	02/01/1996 - 31/07/2015
S&P500 index price	02/01/1986 - 31/07/2015

Note – Data sources. The daily CAD/USD, CAD/GBP and CERI are from Statistics Canada. The WTI crude oil price and Brent crude oil price are from Energy Information Administration. The daily Canadian 3-month Treasury bill rate (CTB3) is from the Bank of Canada. 5-minute data on CAD/USD, WTI crude oil price and S&P500 index price are from CQG data factory [http://www.cqg.com]. The daily AUD/USD, AUD/JPY and WTI are from the Reserve Bank of Australia. The daily gold price is the London pm fixing from the London Bullion Market Association (LBMA). Daily NOK/USD, NOK/GBP and Norwegian 3-month effective synthetic rates (NESR3) are from the Norges Bank. Daily CLP/USD is from the Central Bank of Chile. The daily copper price is the London Metal Exchange (LME) price obtained from the Chilean Copper Commission and Quandl. The daily S&P500 composite index closing price comes from Yahoo Finance.

interest rates (i) over the period 1986-2015.[5] The nominal exchange rates includes six bilateral rates expressed as a number of domestic currency units per foreign currency (CAD/USD, CAD/GBP, AUD/USD, AUD/JPY, NOK/USD and CLP/USD), and two effective exchange rates: the Canadian-dollar effective exchange-rate index (CERI), and the Australian-dollar trade-weighted index (TWI). Four commodities (West Texas Intermediate (WTI) crude oil, Brent crude oil, gold and copper) are all priced in U.S. dollars. We use the daily closing level of the S&P 500 index as an auxiliary variable, because it is an indicator of the general level of asset prices, which may have predictive power for both commodity prices and exchange rates. We also consider two short-term interest rates as auxiliary variables: the Canadian 3-month treasury bill rate (CTB3), and the Norwegian 3-month effective synthetic rate (NESR3). Further, we examine the case of the Canadian dollar at 5-minute frequency over 2005-2009. Data descriptions, notation and sources are displayed in Table 1.

As already noted, we use the price of a single dominant exporting commodity for each country instead of the price of a country-specific commodity index. We focus on four typical pairs of commodity prices and exchange rates: the Australian dollar and the price of gold, Canadian dollar and the WTI crude oil price, Norwegian krone and the Brent oil price, and Chilean peso and the

[5] Results on the daily data over the period 2000-2009 are available in an earliwer version of this paper [Zhang, Dufour and Galbraith (2013)]. They are qualitively similar to those presented here.

price of copper.

We perform standard augmented Dickey-Fuller tests on the logarithms of the exchange rate, commodity price and S&P500 price (denoted by lower case, e.g. e, p_{com} and p_{sp}) and their first differences (denoted using Δ, e.g. Δe, Δp_{com} and Δp_{sp}). The results (not reported) suggest that these variables in the logarithms level may be non-stationary, and that the corresponding first differences are all stationary. We therefore model the first difference following a logarithmic transformation in each case, as VAR(k) model,

$$W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t) \tag{3.1}$$

where $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t))'$ for unconditional causality; for conditional causality, $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t), \Delta p_{\text{sp}}(t))'$ or $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t), i(t))'$; and the value of k is chosen according to the Akaike information criterion (AIC). Given this model, we first test the null hypothesis of non-causality between the exchange rate and commodity price for each country at horizon one (one-day ahead), without or with an auxiliary variable. For example, for testing the null hypothesis that WTI oil price does not cause CAD/USD unconditionally at horizon one [denoted as H_0: WTI oil $\nrightarrow$ CAD/USD], a Wald-type test can be applied to test the restriction of $[\pi_1]_{12} = [\pi_2]_{12} = \ldots = [\pi_k]_{12} = 0$ in model (3.1), where $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{\text{oil}}(t))'$. The $p-$values of non-causality tests will be reported in tables. To compare the strength of causality in different directions across multiple horizons, we then apply the method introduced in Section 2 to estimate unconditional and conditional causality measures up to horizon ten (ten-day ahead) and build bootstrap confidence intervals. The results of causality measures will be presented in figures for the convenience of comparison.

In the following sub-sections we report and discuss the results for U.S. dollar-denominated exchange rates, and then two robustness checks: non-U.S. dollar-denominated exchange rates, and the use of interest rates rather than equity prices as conditioning variables.

3.2. U.S. dollar-denominated exchange rates

We first report and discuss the empirical results concerning Granger non-causality tests at horizon one, and multi-horizon causality measures for U.S.- dollar-denominated exchange rates (*i.e.*, exchange rates based on the U.S. dollar as numeraire, so that, for example, the CAD/USD is the exchange-rate measure taken for Canada). The resulting p-values for unconditional and conditional Granger non-causality tests at horizon one for these exchange rates appear in Tables 2 and 3. In the figures we present a large set of results on measurement of the strength of causality in order to uncover broad patterns present in the data, and we summarize these patterns in the text.

The results of the unconditional non-causality tests in Table 2 show that commodity to currency non-causality is strongly rejected in all cases. This is true in cases with conditioning on the S&P500 index level (Table 3). Results on currency to commodity non-causality are mixed: $p-$values are typically bigger than or in the neighborhood of the conventional 0.05 level, but are significant only in the Chilean case at this standard level.

While the Granger-non-causality test results provide strong evidence of effects in the direction

Table 2: Unconditional Granger non-causality tests at horizon one
(CAD/USD, AUD/USD, NOK/USD and CLP/USD)

Canada (03/01/1986 - 31/07/2015)		Australia (03/01/1986 - 31/07/2015)	
WTI oil $\nrightarrow$ CAD/USD	0.000	gold $\nrightarrow$ AUD/USD	0.000
CAD/USD $\nrightarrow$ WTI oil	0.082	AUD/USD $\nrightarrow$ gold	0.053
Norway (21/05/1987 - 31/07/2015)		Chile (03/01/1996 - 31/07/2015)	
Brent oil $\nrightarrow$ NOK/USD	0.000	copper $\nrightarrow$ CLP/USD	0.000
NOK/USD $\nrightarrow$ Brent oil	0.127	CLP/USD $\nrightarrow$ copper	0.022

Note – Tests are based on model (3.1) where $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t))'$. WTI oil $\nrightarrow$ CAD/USD denotes the null hypothesis of unconditional Granger non-causality at horizon one from WTI oil price to CAD/USD. The other notations are analogous. $P-$values of tests are reported in the table.

of commodity price to exchange rate at horizon one, there is some evidence of effects in the other direction as well. However, these tests are restricted to horizon one, and do not provide measures of the strength of causality between variables. In cases where we reject the non-causality hypothesis in both directions, the tests may mask the potential difference in the strength of these effects; in some cases, causality may be very weak even if non-causality is rejected. We therefore turn next to measures of the magnitudes of these effects across multiple horizons, computing the causality measures using the methods described in Section 2. The results are reported primarily through graphics.

The unconditional U.S. dollar-denominated causality measures are reported in the left columns of Figures 1 - 4, and the conditional causality measures are reported in the right columns of these figures, in each case up to a ten-period horizon. A causality measure is statistically significant when the confidence interval does not include the value zero; for example, from the top left panel of Figure 1, we can conclude that crude oil has significant predictive power for the CAD/USD exchange rate up to 3 days. In reading the figures, note that vertical scales may differ; to facilitate comparisons we have therefore included a number of panels in which effects in the two directions are recorded on a common scale.

We note a few broad patterns that are observable in the figures: (1) causality measures usually have the highest value at horizon one and decrease with increasing prediction horizon, and tend to converge toward zero with increasingly tight confidence intervals; (2) in cases where non-causality is rejected in both directions, causality measures in the two directions can typically be distinguished to some extent; (3) in cases where the non-causality hypothesis is not rejected, we find the corresponding measures are low but still statistically significant, which may indicate that causality measures provide a more powerful way to test Granger non-causality.

In the unconditional cases, we observe (see the bottom left panels of Figures 1 - 4) that causality running from commodity to currency is stronger than in the opposite direction at horizon one in all cases; thereafter the effects drop off rapidly and are not clearly distinguishable. In particular, the

Table 3: Conditional Granger non-causality tests at horizon one - conditional on S&P500 price (CAD/USD, AUD/USD, NOK/USD and CLP/USD)

Canada (03/01/1986 - 31/07/2015)		**Australia** (03/01/1986 - 31/07/2015)	
WTI oil $\nrightarrow$ CAD/USD | S&P500	0.001	Gold $\nrightarrow$ AUD/USD | S&P500	0.000
CAD/USD $\nrightarrow$ WTI oil | S&P500	0.042	AUD/USD $\nrightarrow$ Gold | S&P500	0.055
Norway (21/05/1987 - 31/07/2015)		**Chile** (03/01/1996 - 31/07/2015)	
Brent oil $\nrightarrow$ NOK/USD | S&P500	0.000	Copper $\nrightarrow$ CLP/USD | S&P500	0.000
NOK/USD $\nrightarrow$ Brent oil | S&P500	0.065	CLP/USD $\nrightarrow$ Copper | S&P500	0.013

Note – Tests are based on model (3.1) where $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t), \Delta p_{\text{sp}}(t))'$. WTI oil $\nrightarrow$ CAD/USD | S&P500 denotes the null hypothesis of Granger non-causality at horizon one from WTI oil price to CAD/USD conditional on S&P500 price. The other notations are analogous. $P-$values of tests are reported in the table.

ratio of causality measures in the two directions at horizon one can be quite high, for example, as high as 3 for Canada, 10 for Australia and Norway, and over 30 for Chile in favor of causation from commodity price to exchange rate.

The lesson is essentially the same in conditional cases, conditioning here on the S&P index value. The bottom right panels of Figures 1 - 4 provide summary results of conditional cases: the strongest effects are measured at horizon one, where the commodity-to-currency direction dominates in all cases.

Globally, these results suggest stronger causation from commodity price to exchange rate rather than vice versa. We will now examine whether the elimination of dollar effects or conditioning on interest rates has an effect on the overall pattern.

3.3. Robustness checks

We first check robustness to the use of non-U.S. dollar-denominated exchange rates (alternative currency benchmarks and the effective exchange rates) for Canada and Australia, using GBP-denominated Canadian dollar (GBP/CAD), the Canadian effective exchange-rate index (CERI), JPY-denominated Australian dollar (AUS/JPY), and the Australian trade weighted exchange-rate index (TWI). Adequate data with which to check robustness to the use of short-term interest rates rather than the level of equity prices as conditioning variables, are available only for Canada and Norway: the Canadian 3-month treasury bill rate (CTB3) and the Norwegian 3-month effective synthetic rate (NESR3). We also examine the use of 5-minute data, available for oil prices, the CAD/USD exchange rate and S&P500 index price for a shorter sample.

Tables 4 and 5 give unconditional and conditional statistical inference for the alternative currency benchmarks. The results are qualitatively similar to those for the U.S. dollar-denominated currencies, with one noteworthy exception: evidence of commodity to currency causality does not appear in the case of the Canadian dollar exchange rate with respect to Sterling. This result can

Table 4: Unconditional Granger non-causality tests at horizon one
(CAD/GBP, CERI, AUD/JPY and AUD(TWI))

Canada (03/01/1986 - 31/07/2015)			
WTI oil $\nrightarrow$ CAD/GBP	0.295	WTI oil $\nrightarrow$ CERI	0.000
CAD/GBP $\nrightarrow$ WTI oil	0.847	CERI $\nrightarrow$ WTI oil	0.107
Australia (03/01/1986 - 31/07/2015)			
Gold $\nrightarrow$ AUD/JPY	0.000	Gold $\nrightarrow$ AUD(TWI)	0.000
AUD/JPY $\nrightarrow$ Gold	0.363	AUD(TWI) $\nrightarrow$ Gold	0.304

Note – Tests are based on model (3.1) where $W(t) = (\Delta e(t), \Delta p_{\mathrm{com}}(t))'$. WTI oil $\nrightarrow$ CAD/GBP denotes the null hypothesis of unconditional Granger non-causality at horizon one from WTI oil price to CAD/GBP. The other notations are analogous. $P-$values of tests are reported in the table.

Table 5: Conditional Granger non-causality tests at horizon one - conditional on S&P500 price
(CAD/GBP, CERI, AUD/JPY and AUD(TWI))

Canada (03/01/1986 - 31/07/2015)			
WTI oil $\nrightarrow$ CAD/GBP $\mid$ S&P500	0.612	WTI oil $\nrightarrow$ CERI $\mid$ S&P500	0.002
CAD/GBP $\nrightarrow$ WTI oil $\mid$ S&P500	0.928	CERI $\nrightarrow$ WTI oil $\mid$ S&P500	0.092
Australia (03/01/1986 - 31/07/2015)			
Gold $\nrightarrow$ AUD/JPY $\mid$ S&P500	0.000	Gold $\nrightarrow$ AUD(TWI) $\mid$ S&P500	0.000
AUD/JPY $\nrightarrow$ Gold $\mid$ S&P500	0.706	AUD(TWI) $\nrightarrow$ Gold $\mid$ S&P500	0.188

Note – Tests are based on model (3.1) where $W(t) = (\Delta e(t), \Delta p_{\mathrm{com}}(t), \Delta p_{\mathrm{sp}}(t))'$. WTI oil $\nrightarrow$ CAD/GBP $\mid$ S&P500 denotes the null hypothesis of Granger non-causality at horizon 1 from WTI oil price to CAD/GBP conditional on S&P500 price. The other notations are analogous. $P-$values of tests are reported in the table.

Table 6: Conditional Granger non-causality tests at horizon one - conditional on interest rate (CAD/USD and NOK/USD)

Canada (03/01/1986 - 31/07/2015)	
WTI oil $\nrightarrow$ CAD/USD | CTB3	0.001
CAD/USD $\nrightarrow$ WTI oil | CTB3	0.102
Norway (08/01/2003 - 31/07/2015)	
Brent oil $\nrightarrow$ NOK/USD | NESR3	0.000
NOK/USD $\nrightarrow$ Brent oil | NESR3	0.101

Note – Tests are based on model (3.1) where $W(t) = (\Delta e(t), \Delta p_{\text{com}}(t), i(t))'$. WTI oil $\nrightarrow$ CAD/USD | CTB3 denotes the null hypothesis of Granger non-causality at horizon one from WTI oil price to CAD/USD conditional on CTB3. The other notations are analogous. $P-$values of tests are reported in the table.

be seen as a form of falsification test: Sterling was itself a heavily commodity influenced currency through a large part of this sample period, when North Sea oil sales were at their peak; oil prices may therefore have be expected to affect the two currencies in a similar way, so their relative value is not greatly affected.

For the other non-U.S. dollar-denominated exchange rates, as well as the effective exchange rates, we find evidence of causality running from commodity prices to exchange rates, but not for the opposite direction, in both unconditional and conditional analyses.

To assess the strength of causality under the alternative exchange-rate benchmark, compare Figure 1 with Figures 7 and 8 (Canada) and Figure 2 with Figures 9 and 10 (Australia). Comparing Figure 1 with Figure 8, we see nearly identical results, since the weight of the U.S. dollar in the Canadian dollar effective rate is over 80%. This is not the case when comparing Figure 1 with Figure 7, in which we treat the CAD exchange rate relative to GBP. Then, as indicated above, the effect of oil price changes is expected to be similar for both the Canadian and British currencies, leading to reduced observable effect of the oil price on this particular exchange rate. But the reverse effect drops even more: the causality measure from the gold price to CAD/GBP is stronger than that in the opposite direction, across multiple horizons.

For the Australian data, comparing Figure 2 with Figures 9 and 10, we observe similar patterns: causality from commodity price to exchange rate is stronger than the opposite direction up to horizon ten, though the magnitudes of the effects are lower in the latter cases.

Table 6 provides statistical evidence for Canada and Norway, where we use a short-term (three-month) interest rate as conditioning variable rather than the level of equity prices (the S&P500 index price). As in previous cases, we can only reject the null hypothesis of non-causality from commodity to currency, but not in the other direction.

The effect on causality measures of this change of conditioning variable (*i.e.*, replacing the equity price variable with short-term interest rates) can be observed in Figures 5 and 6 for Canada and Norway respectively. On comparing two columns of Figure 5, we see that the results are qualitatively the same, but including the short-term interest rate increases the magnitude of causality from the exchange rate to the commodity price. From the comparison of the right column with the left one of Figure 6, we see that the patterns are qualitatively similar with slightly larger magnitudes. The results also suggest that the interest rate helps to identify a causation from exchange rates to commodity price across multiple horizons, although it is weak. Overall, these findings are consistent with our previous conclusion: causality from commodity price to exchange rate is stronger than in the opposite direction, especially at short horizons.

A final robustness check involves the use of 5-minute data on oil prices and exchange rates, available for Canada only. This sample extends from 03/01/2005 to 31/12/2009. The causality measures are presented in Figure 11, which is similar to Figure 1 but at the 5-minute frequency. The 5-minute results qualitatively agree with the daily ones. There is weak evidence of Granger-causality in both directions, but it is stronger in the direction of commodity price to exchange rate, at horizon one. The measures drop quickly after horizon one, and the differences between measures in the two directions are small.

Globally, the above sensitivity analysis corroborates the conclusions obtained in the previous subsection: Granger-causality from commodity prices to exchange rates is much stronger than for the opposite direction, especially at horizon one; it is weaker in both directions at other horizons. Statistical inference is compatible with this conclusion, providing very strong evidence of Granger-causality from commodity prices to exchange rates, but mixed evidence of any causal effect in the opposite direction.

4. Conclusion

Both popular commentary and economic reasoning based on demand for the currencies of small open economies suggest that causality should run from commodity prices to exchange rates, but the present value model of forward-looking exchange rates implies that exchange rates should Granger-cause commodity prices. The debate on the direction of causality between commodity prices and exchange rates is still open. Here we have examined higher-frequency causal relationships between exchange rates of four typical commodity economies (Canada, Norway, Australia, and Chile) and the prices of their corresponding dominant exporting commodities (crude oil, gold, and copper). We use daily and 5-minute data to reduce time-aggregation effects. In addition, we have applied the concept of multi-horizon causality measures to compare the strength of causal relationships, to provide more powerful non-causality tests, and to determine how long the causal effects will last.

In contrast with previous studies, our results suggest that unconditional and conditional causality running from commodity prices to exchange rates is stronger than that in the opposite direction across multiple horizons. In more detail, we find that (1) there is evidence of Granger causality between commodity prices and exchange rates in both directions across multiple horizons, but the evidence and measured strength are much stronger in the direction of commodity price to exchange rate (the macroeconomic/trade mechanism), especially at short horizons; (2) causality is stronger at short horizons, and becomes weaker as the horizon increases; (3) conditioning on S&P500 price does not change the patterns of causality measures found in unconditional cases. (4) eliminating dollar effects tends to weaken further the evidence of causality from exchange rates to commodity prices, and reveals a more definite pattern where causality from commodity prices to exchange rates is stronger than causality in the reverse direction across multiple horizons; and (5) the main results are robust to including interest rates as conditional variables.

These results suggest that the macroeconomic/trade-based mechanism mentioned in the introduction plays a central role in exchange-rate dynamics, despite the financial features of these markets. To "see" these effects in the data, it is however important to consider a sufficiently high data frequency and to use an appropriate causal methodology. The results also underscore the facts that the interpretation of causality depends on time units and observation intervals (data frequency), and that causality measures present a more informative analysis of Granger causality than tests of non-causality alone.

High-frequency data are potentially very fruitful in causality studies, allowing us to distinguish with high resolution between immediate and lagged effects corresponding with different agents' interests. However, there remain further avenues to investigate. For example, in our causality measures with 5-minute data, we estimate the VAR model at this frequency and the causality measures lasting up to 11 periods, that is, only about one hour. If we were to allow longer periods for the effects to develop we would need a large number of lags in the VAR model, sacrificing estimation efficiency. One possible method of handling this difficulty is to use mixed-data sampling (MIDAS) and mixed-frequency VAR (MF- VAR) approaches [Ghysels, Santa-Clara and Valkanov (2004), Ghysels, Sinko and Valkanov (2007), Ghysels, Hill and Motegi (2013) and Kuzin, Marcellino and Schumacher (2010)]. Furthermore, it is interesting to consider out-of-sample tests for Granger causality [Inoue and Kilian (2004) and Chen (2005)]. Another worthwhile extension would be to examine causality between volatility of commodity prices and exchange rates using the concept of second-order causality [Granger, Robins and Engle (1986), Comte and Lieberman (2000), Hafner (2009), and Dufour and Zhang (2015)].

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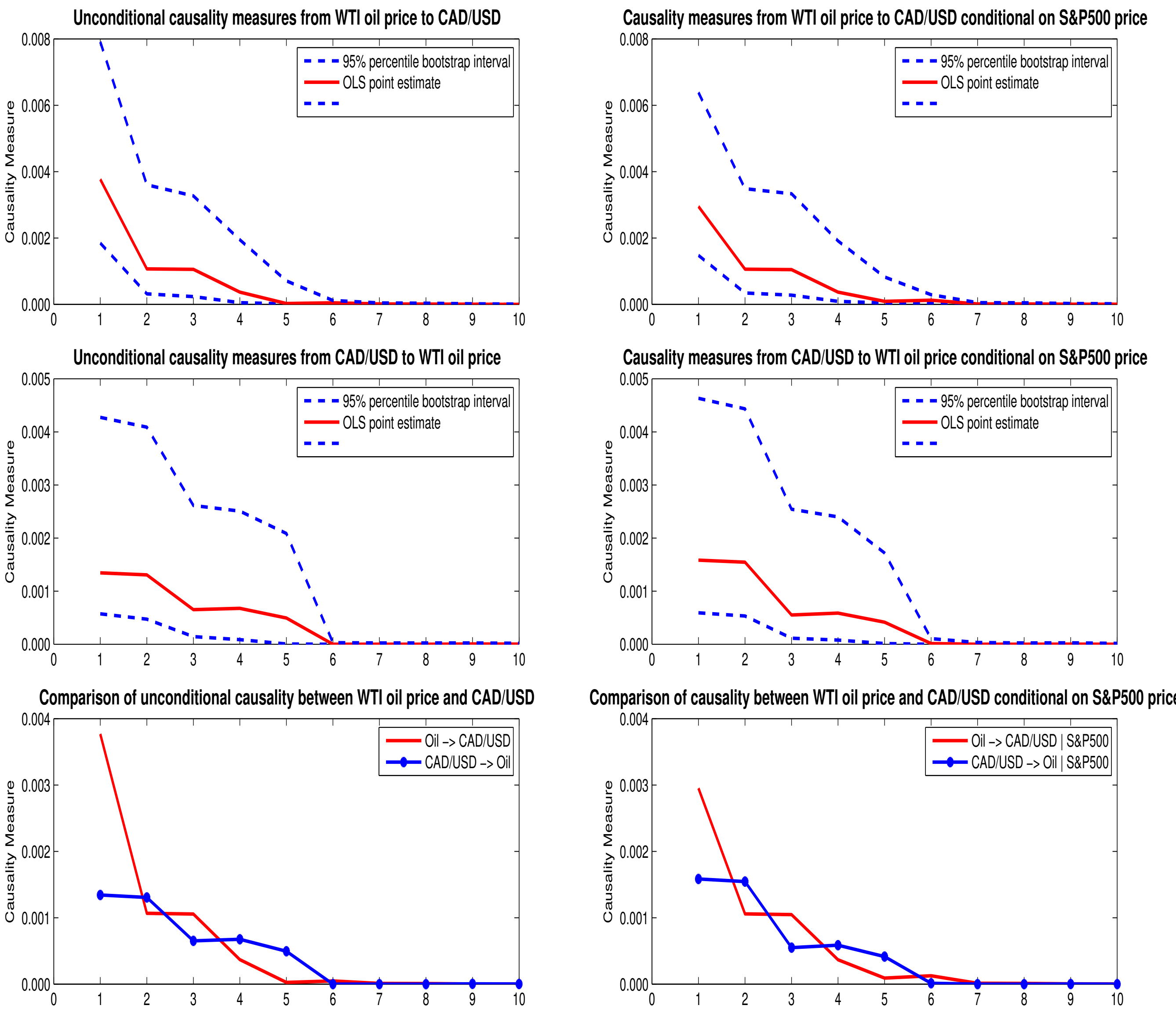
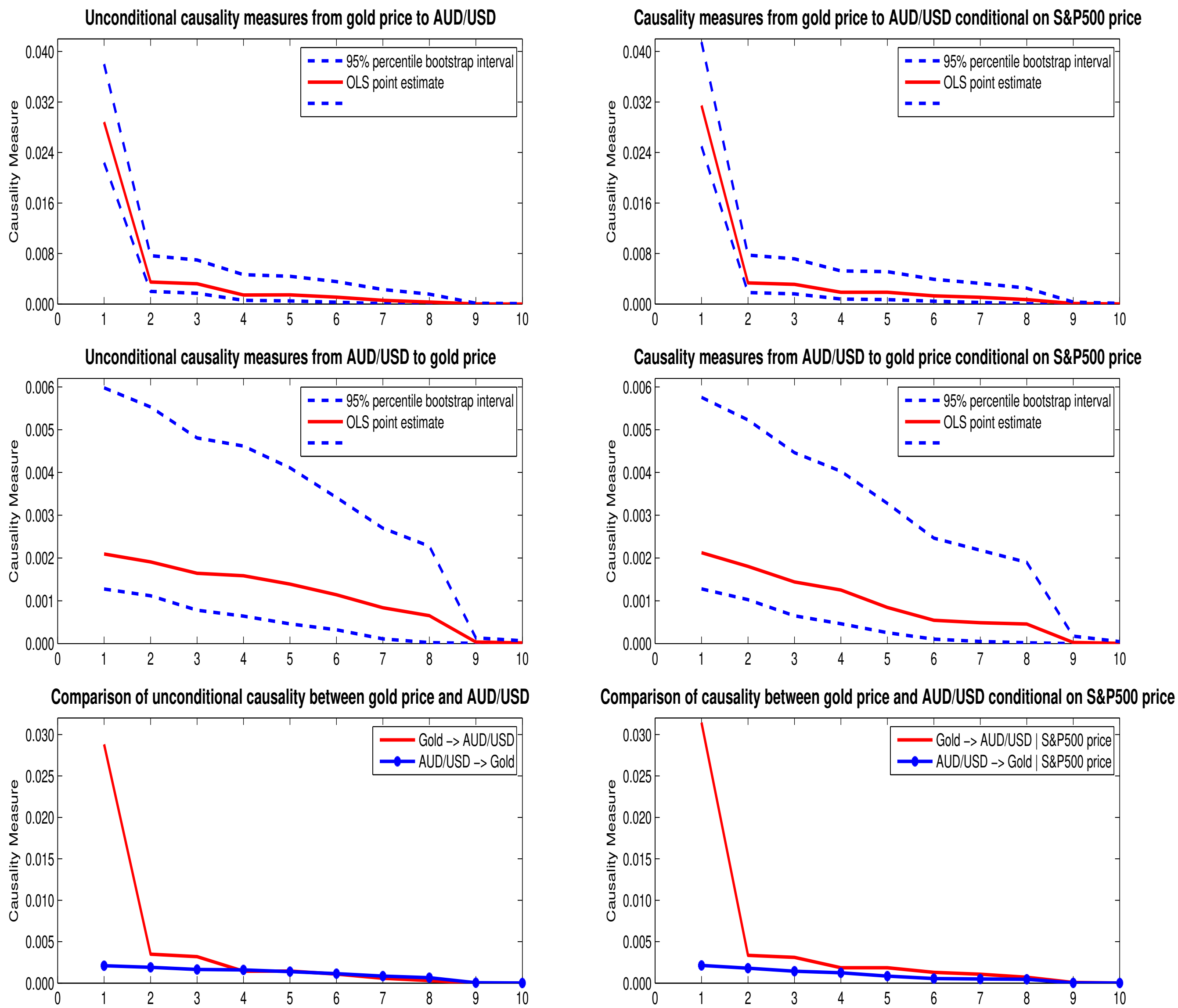
Figure 1. Causality measures between CAD/USD and WTI oil price
(unconditional and conditional on S&P500 price)



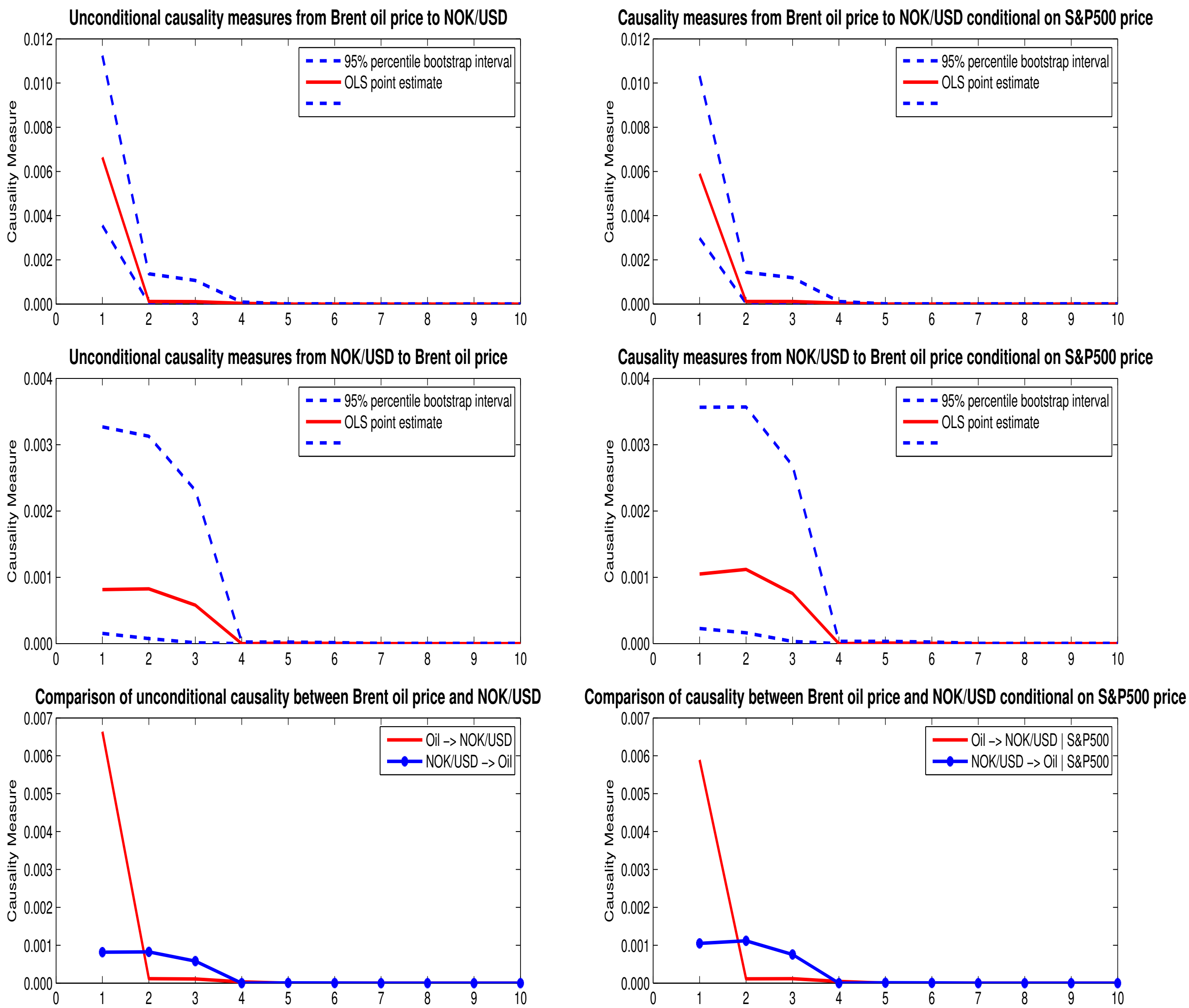
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t))'$ and $k = 5$.
  - $*$ Conditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t), \Delta p_{sp}(t))'$ and $k = 5$.
- Data: Daily CAD/USD, WTI oil price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 2. Causality measures between AUD/USD and gold price
(unconditional and conditional on S&P500 price)



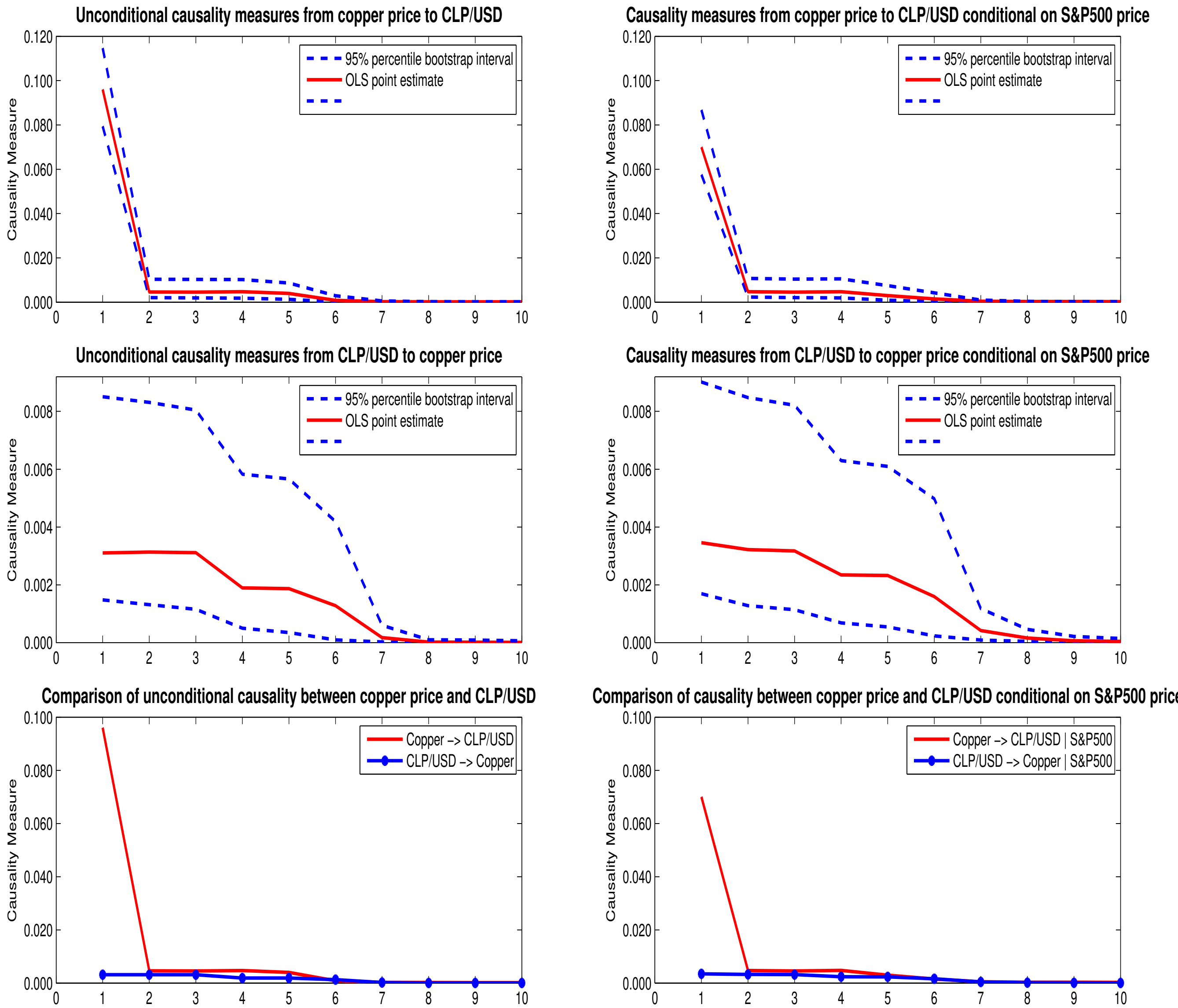
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{AUD/USD}(t), \Delta p_{gold}(t))'$ and $k = 8$.
  - $*$ Conditional: $W(t) = (\Delta e_{AUD/USD}(t), \Delta p_{gold}(t), \Delta p_{sp}(t))'$ and $k = 8$.
- Data: Daily AUD/USD, gold price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 3. Causality measures between NOK/USD and Brent oil price
(unconditional and conditional on S&P500 price)



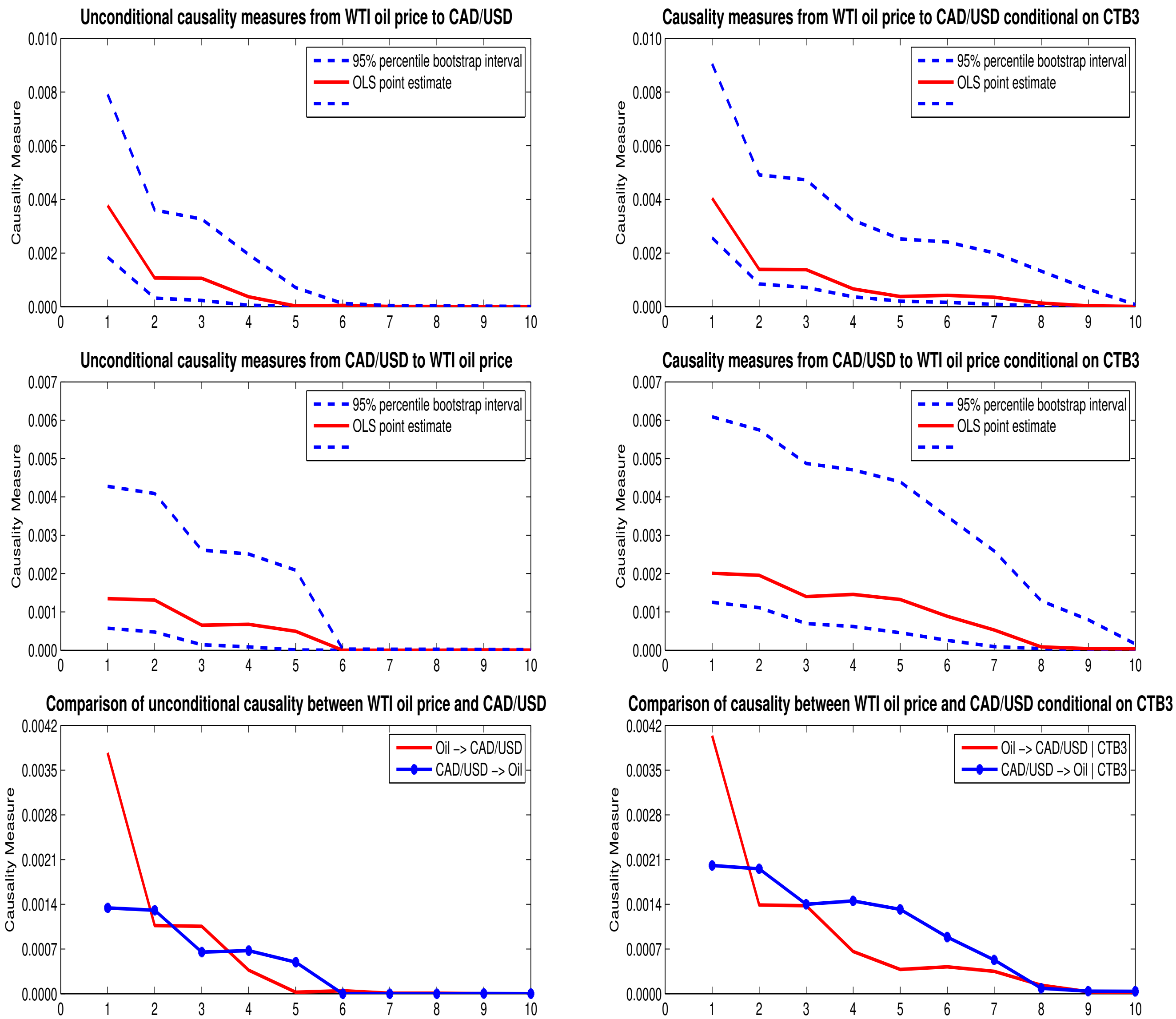
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$
  - $*$ Unconditional: $W(t) = (\Delta e_{NOK/USD}(t), \Delta p_{oil}(t))'$ and $k = 3$.
  - $*$ Conditional: $W(t) = (\Delta e_{NOK/USD}(t), \Delta p_{oil}(t), \Delta p_{sp}(t))'$ and $k = 3$.
- Data: Daily NOK/USD, Brent oil price and S&P500 price are from 20/05/1987 to 31/07/2015.

Figure 4. Causality measures between CLP/USD and copper price
(unconditional and conditional on S&P500 price)



- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$
  - $*$ Unconditional: $W(t) = (\Delta e_{CLP/USD}(t), \Delta p_{copper}(t))'$ and $k = 6$.
  - $*$ Conditional: $W(t) = (\Delta e_{CLP/USD}(t), \Delta p_{copper}(t), \Delta p_{sp}(t))'$ and $k = 6$.
- Data: Daily CLP/USD, copper price and S&P500 price are from 03/01/1996 to 31/07/2015.

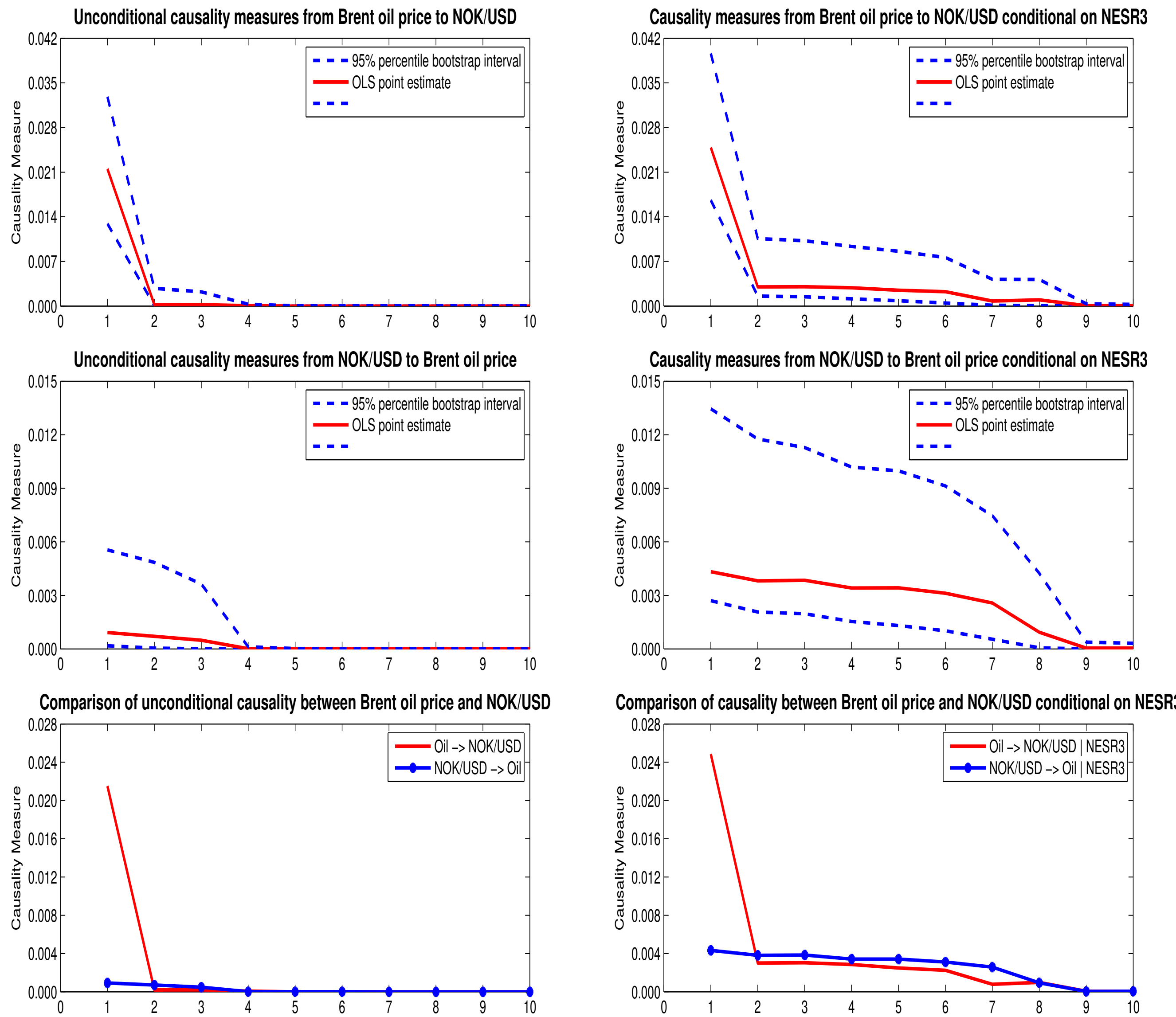
Figure 5. Causality measures between CAD/USD and WTI oil price
(unconditional and conditional on CTB3)



- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t))'$ and $k = 9$.
  - $*$ Conditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t), i_{CTB3}(t))'$ and $k = 9$.

- Data: Daily CAD/USD, WTI oil price and Canadian 3-month Treasury bill rate (CTB3) are from 03/01/1986 to 31/07/2015.

Figure 6. Causality measures between NOK/USD and Brent oil price
(unconditional and conditional on NESR3)



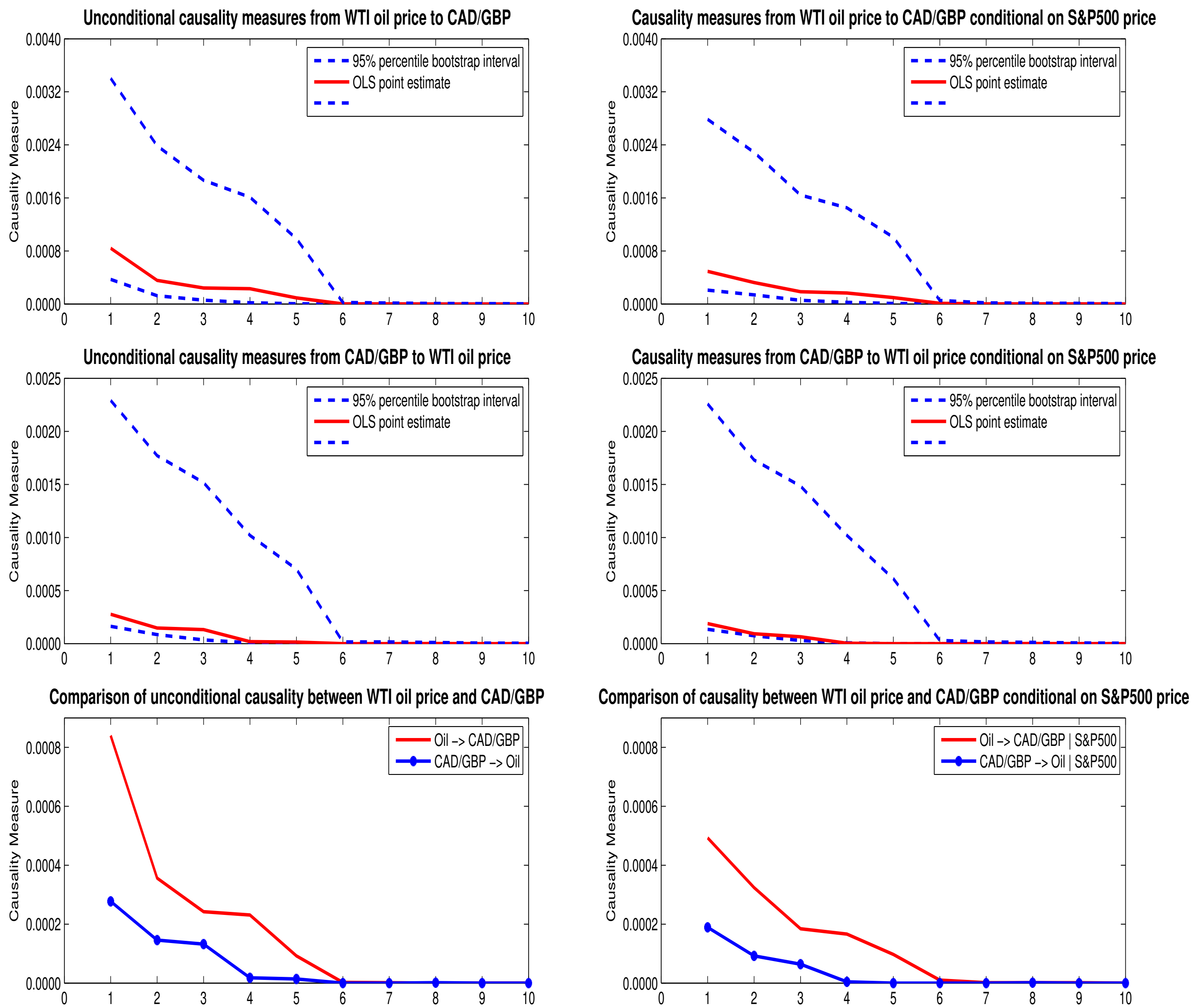
• Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$

  ∗ Unconditional: $W(t) = (\Delta e_{NOK/USD}(t), \Delta p_{oil}(t))'$ and $k = 8$.

  ∗ Conditional: $W(t) = (\Delta e_{NOK/USD}(t), \Delta p_{oil}(t), i_{NESR3}(t))'$ and $k = 8$.

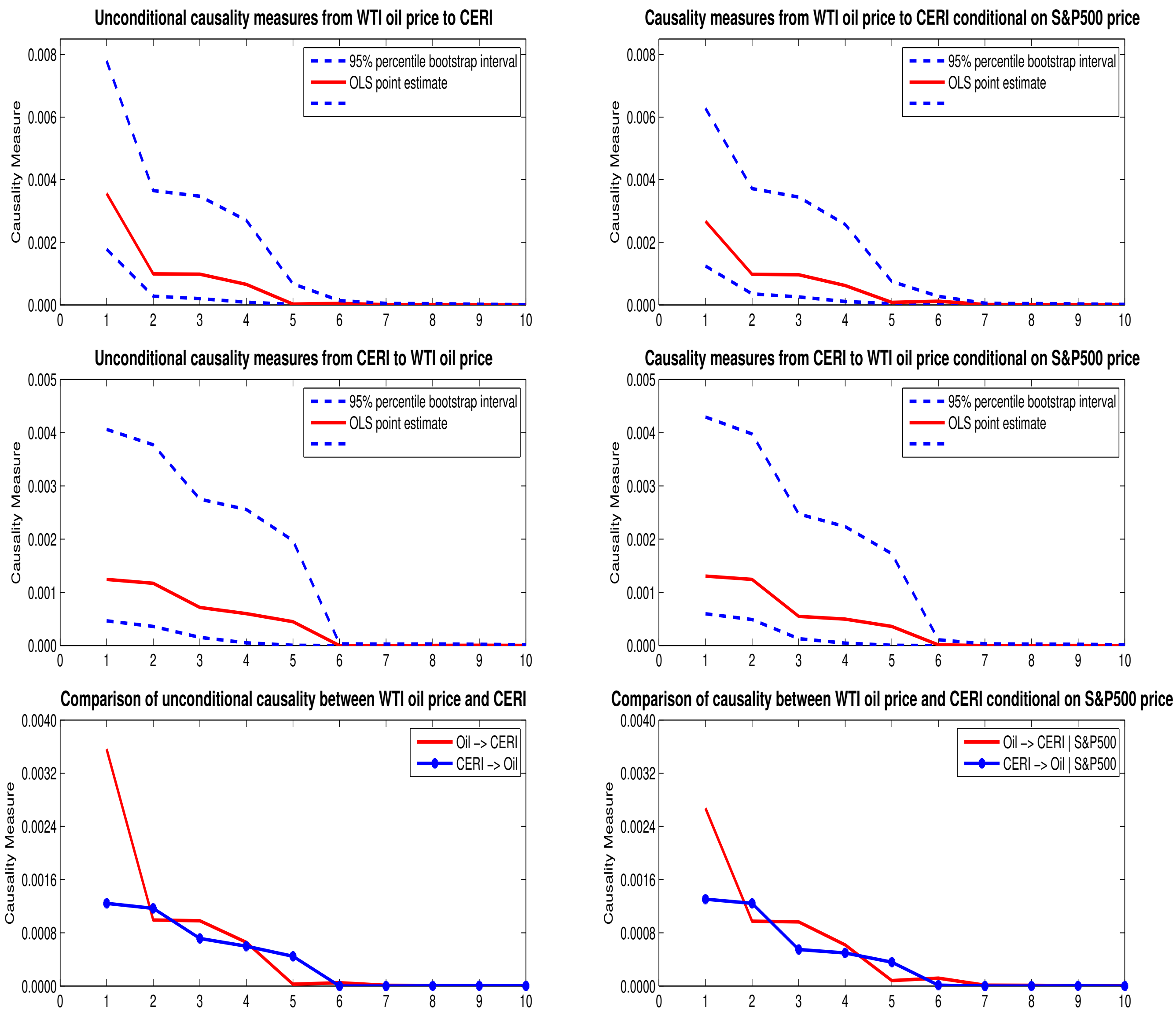
• Data: Daily NOK/USD, Brent oil price and Norwegian 3-month effective synthetic rate (NESR3) are from 08/01/2003 to 31/07/2015.

Figure 7. Causality measures between CAD/GBP and WTI oil price
(unconditional and conditional on S&P500 price)



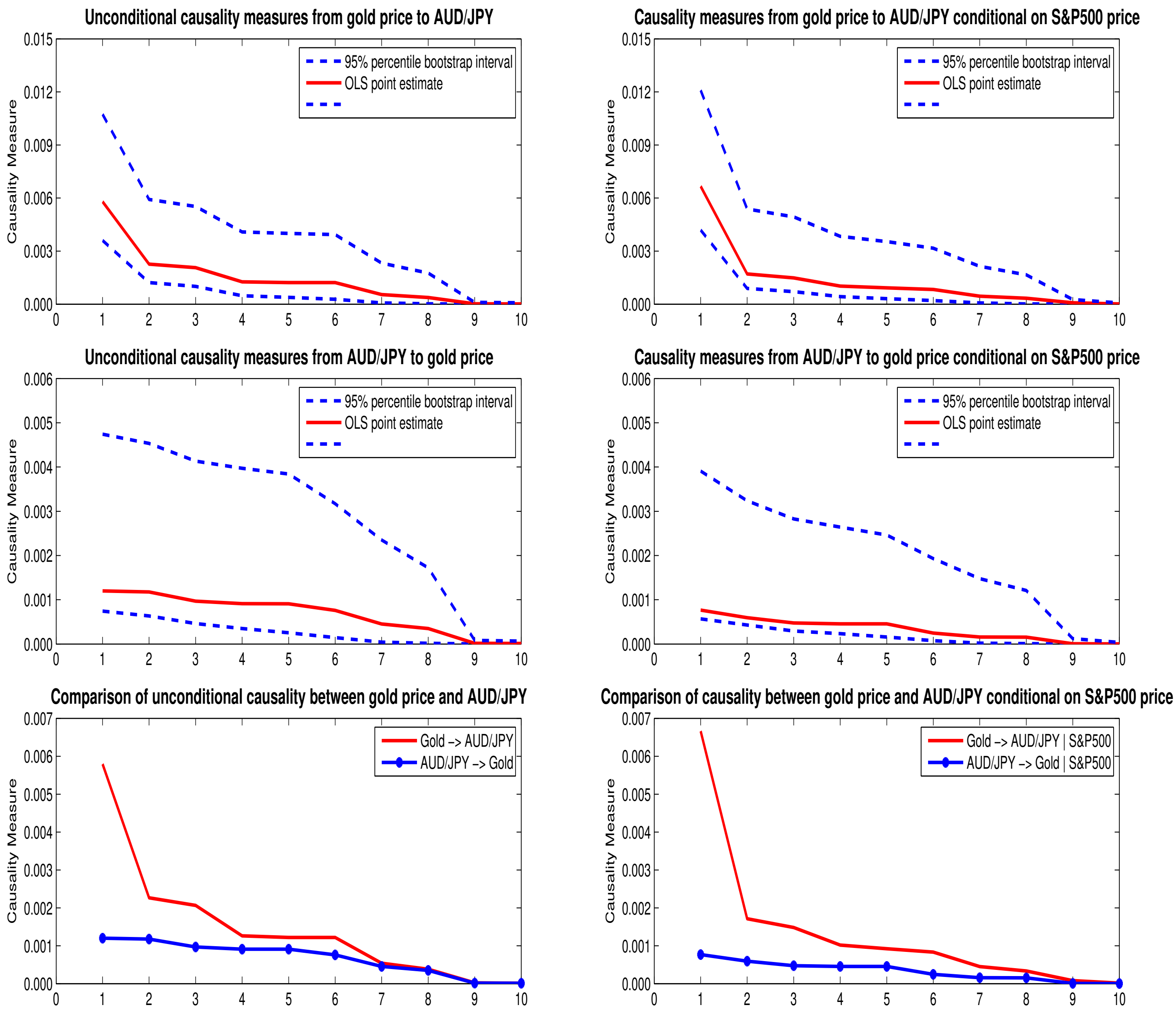
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{CAD/GBP}(t), \Delta p_{oil}(t))'$ and $k = 5$.
  - $*$ Conditional: $W(t) = (\Delta e_{CAD/GBP}(t), \Delta p_{oil}(t), \Delta p_{sp}(t))'$ and $k = 5$.
- Data: Daily CAD/GBP, WTI oil price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 8. Causality measures between CERI and WTI oil price
(unconditional and conditional on S&P500 price)



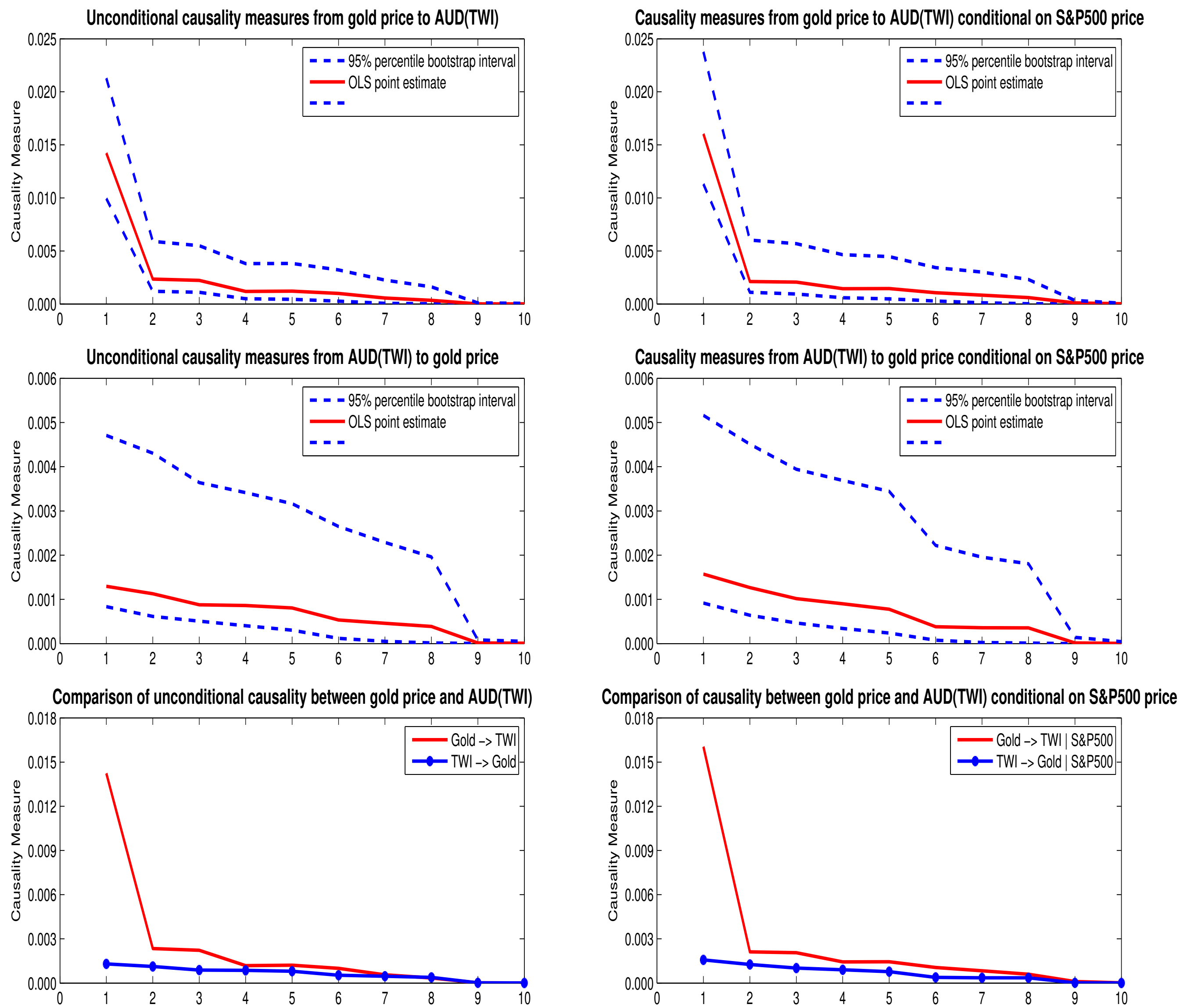
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{CERI}(t), \Delta p_{oil}(t))'$ and $k = 5$.
  - $*$ Conditional: $W(t) = (\Delta e_{CERI}(t), \Delta p_{oil}(t), \Delta p_{sp}(t))'$ and $k = 5$.
- Data: Daily CERI, WTI oil price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 9. Causality measures between AUD/JPY and gold price
(unconditional and conditional on S&P500 price)



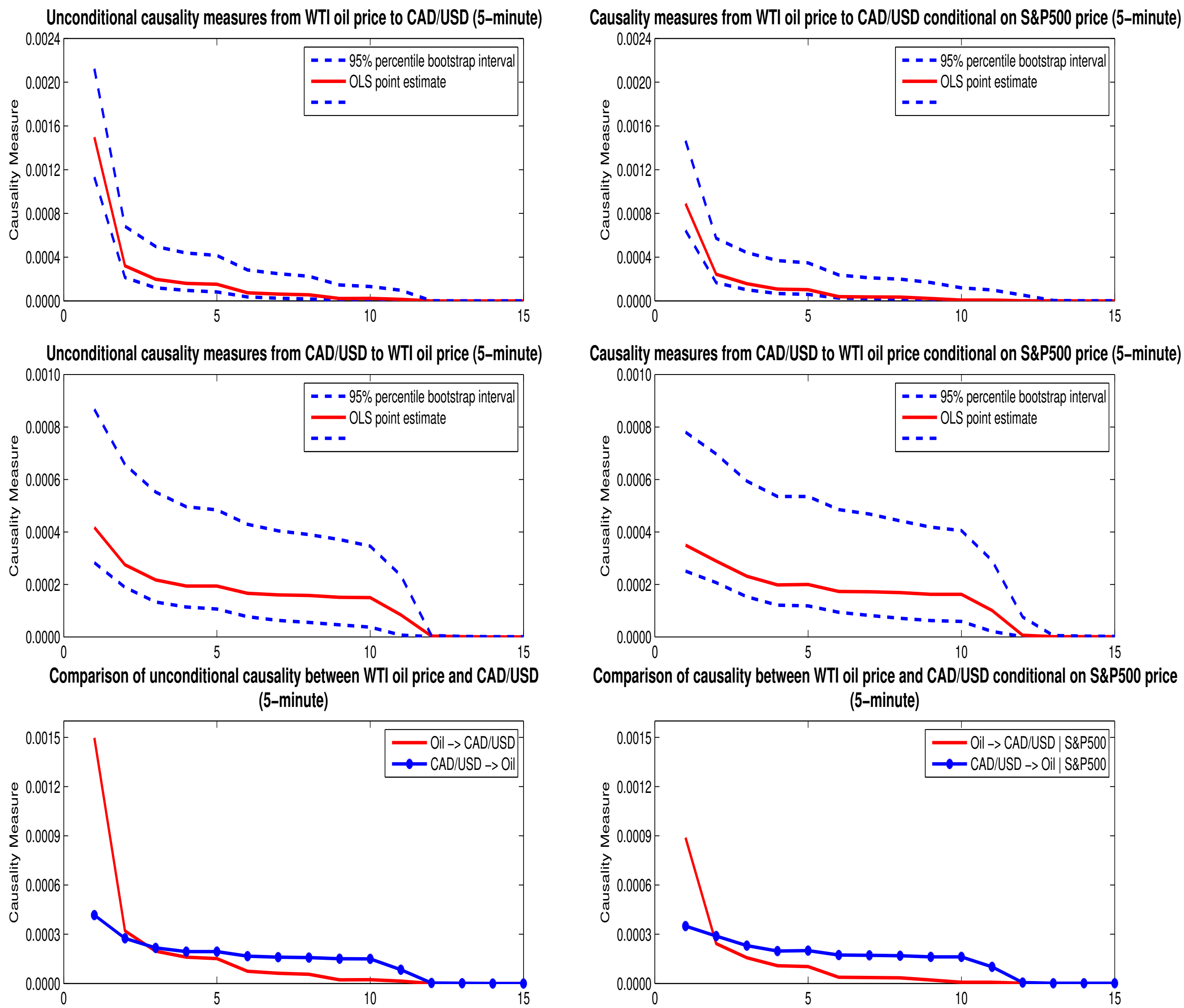
- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{AUD/JPY}(t), \Delta p_{gold}(t))'$ and $k = 8$.
  - $*$ Conditional: $W(t) = (\Delta e_{AUD/JPY}(t), \Delta p_{gold}(t), \Delta p_{sp}(t))'$ and $k = 8$.
- Data: Daily AUD/JPY, gold price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 10. Causality measures between AUD(TWI) and gold price
(unconditional and conditional on S&P500 price)



- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{TWI}(t), \Delta p_{gold}(t))'$ and $k = 8$.
  - $*$ Conditional: $W(t) = (\Delta e_{TWI}(t), \Delta p_{gold}(t), \Delta p_{sp}(t))'$ and $k = 8$.
- Data: Daily AUD (TWI), gold price and S&P500 price are from 03/01/1986 to 31/07/2015.

Figure 11. Causality measures between CAD/USD and WTI oil price at 5-minute
(unconditional and conditional on S&P500 price)



- Model: $W(t) = \pi_0 + \sum_{i=1}^{k} \pi_i W(t-i) + u(t)$.
  - $*$ Unconditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t))'$ and $k = 11$.
  - $*$ Conditional: $W(t) = (\Delta e_{CAD/USD}(t), \Delta p_{oil}(t), \Delta p_{sp}(t))'$ and $k = 11$.
- Data: 5-minute CAD/USD, WTI oil price and S&P500 price are from 03/01/2005 to 31/12/2009.