Jean-Marie Dufour
February 18, 2019

McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

25 points

1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

   (a) If a random variable has finite second moments, it has finite moments at all higher orders.

   (b) Any stationary process of order 5 is also stationary of order 2.

   (c) Any strictly stationary process is in L_2.

   (d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.

   (e) Non-invertible moving processes have no covariance generating function.

50 points

2. Consider the following models:

$$X_t = 10 + 0.7\, X_{t-1} - 0.2\, X_{t-2} + u_t \tag{1}$$

   where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. For each one of these models, answer the following questions.

   (a) Is this model stationary? Why?

   (b) Is this model invertible? Why?

   (c) Compute:

      i. $E(X_t)$;

      ii. $\gamma(k)$, $k = 1, \ldots, 8$;

      iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Find the autocovariance generating function of X_t.

(g) Compute the first two partial autocorrelations of X_t.

(h) If $X_{10} = 11$, compute the best linear forecast of X_{11} based on X_{10} (only). Justify your answer.

(i) If $X_8 = 12$, $X_9 = 9$ and $X_{10} = 11$, compute the best linear forecast of X_{11} and X_{12} based on the past X_t up to time 10. Justify your answer.

25 points 3. Let $X_1, X_2, \ldots, X_T$ be a time series.

(a) Define:

i. the sample autocorrelations for this series;

ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

i. under the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.);

ii. under the hypothesis that the process follows a moving average of finite order.

(c) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) without any assumption on the existence of moments for $X_1, X_2, \ldots, X_T$.