Jean-Marie Dufour
February 23, 2016

McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;

(b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;

(c) the function $\gamma(k)$ is positive semi-definite.

20 points 2. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \, , \; t \in \mathbb{Z} \, ,$$

where $\nu_1, \dots, \nu_m$ are distinct constants on the interval $[0,\ 2\pi)$ and A_j, B_j, $j = 1, \dots,$ m, are random variables in L_2 , such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \, , \; E(A_j^2) = E(B_j^2) = \sigma_j^2 \, , \; j = 1, \dots, n \, , \\ E(A_j A_k) &= E(B_j B_k) = 0, \text{ for } j \neq k \, , \\ E(A_j B_k) &= 0, \; \forall j, \; k \, . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic

[Hint: consider the regression of X_t on $\cos(\nu_1 t)$ and $\sin(\nu_1 t)$ based two observations.]

60 points 3. Consider the following models:

$$X_t = 10 + 0.7\, X_{t-1} - 0.2\, X_{t-2} + u_t \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.\ N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;

ii. $\gamma(k)$, $k = 1, \ldots, 8$;

iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Find the autocovariance generating function of X_t.

(g) Compute the first two partial autocorrelations of X_t.

(h) If $X_{10} = 11$, compute the best linear forecast of X_{11} based on X_{10} (only). Justify your answer.

(i) If $X_8 = 12$, $X_9 = 9$ and $X_{10} = 11$, compute the best linear forecast of X_{11} and X_{12} based on the past X_t up to time 10. Justify your answer.