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McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \, , \; t \in \mathbb{Z} \, ,$$

where $\nu_1, \ldots, \nu_m$ are distinct constants on the interval $[0, \, 2\pi)$ and $A_j, B_j, \; j = 1, \ldots,$ m, are random variables in L_2, such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \, , \; E(A_j^2) = E(B_j^2) = \sigma_j^2 \, , \; j = 1, \ldots, n \, , \\ E(A_j A_k) &= E(B_j B_k) = 0, \text{ for } j \neq k \, , \\ E(A_j B_k) &= 0, \; \forall j, \, k \, . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic

[Hint: consider the regression of X_t on $\cos(\nu_1 t)$ and $\sin(\nu_1 t)$ based two observations.]

50 points 2. Consider the following models:

$$X_t = 0.5 \, X_{t-1} + u_t - 0.25 \, u_{t-1} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d. \; N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

   i. $E(X_t)$;
   ii. $\gamma(k)$, $k = 1, \ldots, 8$;
   iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Find the autocovariance generating function of X_t.

(g) Find and graph the spectral density of X_t.

(h) Compute the first two partial autocorrelations of X_t.

30 points 3. Let $X_1, X_2, \ldots, X_T$ be a time series.

(a) Define:

   i. the sample autocorrelations for this series;
   ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

   i. under the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.);
   ii. under the hypothesis that the process follows a moving average of finite order.

(c) Describe how you would identify the process described in equation (1) in question 2.

(d) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) wothout any assumption on the existence of moments for $X_1, X_2, \ldots, X_T$.