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McGill University
ECON 763
Financial econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

30 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly you answer (maximum: one page per statements).

(a) If a stochastic process has finite second moments, it cannot have finite third moments.

(b) The sample autocorrelations of a nonstationary process do not have finite moments.

(c) A strictly stationary process is also weakly stationary.

(d) Convergence in probability does not imply convergence in mean of order two.

(e) The Wold decomposition does not hold for deterministic processes.

(f) The autocorrelation function of a second-order stationary process is positive semidefinite.

(g) The autocorrelations and the partial autocorrelations of a stationary AR(1) process are identical.

(h) The spectral density of a stationary AR(1) model is not constant.

(i) A sign test for the median of independent random variables remains valid if the observations are heteroskedastic.

(j) The normal distribution is not stable.

20 points 2. Consider the following ARMA model:

$$X_t = 0.5\ X_{t-1} + u_t - 0.25\ u_{t-1} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. Answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;

ii. $\gamma(k)\ ,\ k = 1,\ \ldots\ ,\ 8;$

iii. $\rho(k)\ ,\ k = 1,\ 2,\ \ldots\ ,\ 8.$

(d) Graph $\rho(k)\ ,\ k = 1,\ 2,\ \ldots\ ,\ 8.$

(e) Find the coefficients of $u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3}$ and u_{t-4} in the moving average representation of X_t.

(f) Compute the first two partial autocorrelations of X_t.

(g) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{10}, X_{11}, X_{12} and X_{13} based on X_{10} (only)? If so, compute these.

(h) If $X_{10} = 1$, $u_{10} = 2$, $u_9 = 1$, $u_8 = 0.99$, $u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{11}, X_{12} and X_{13} based on the history of the process up to X_{10}? If so, compute these.

15 points 3. Let $X_1,\, X_2,\, \ldots\, ,\, X_T$ be a time series where $X_1,\, X_2,\, \ldots\, ,\, X_T$ have continuous distributions.

(a) Propose a method for testing the hypothesis that $X_1,\, X_2,\, \ldots\, ,\, X_T$ are independent and identically distributed (i.i.d.) without any assumption on the existence of the moments for $X_1,\, X_2,\, \ldots\, ,\, X_T$.

(b) If $X_1,\, X_2,\, \ldots\, ,\, X_T$ have common median m_0, describe a procedure for testing whether these observations are independent without assuming identical distributions.

(c) Consider the "median regression" model:

$$y_t = x_t'\beta + u_t,\ t = 1, \ldots, T, \tag{2}$$

where $x_t,\ t = 1, \ldots, T$, are $k \times 1$ fixed vectors and the the disturbances $u_t,\ t = 1, \ldots, T$, are independent with median zero and continuous distributions. Propose procedures for testing hypotheses of the form $H_0 : \beta = \beta_0$ and build confidence sets for β.

20 points

4. Let $R_{it}, i = 1, \ldots , n,$ be returns on n securities for period t, and $\tilde{R}_{\mathrm{M}t}$ the return on a benchmark portfolio $(t = 1, \ldots , T)$. The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\bar{\mathcal{H}}_{E1} : \bar{a}_i = 0 , \quad i = 1, \ldots , n. \tag{3}$$

in the context of the MLR model

$$r_{it} = \bar{a}_i + \beta_{it}\tilde{r}_{\mathrm{M}t} + \varepsilon_{it} , \quad \beta_{it} = \bar{\beta}_i + \sum_{j=1}^{q} d_{ji} z_{jt} , \tag{4}$$

$t = 1, \ldots , T,\ i = 1, \ldots , n,$ where $r_{it} = R_{it} - R_{ft},\ \tilde{r}_{\mathrm{M}t} = \tilde{R}_{\mathrm{M}t} - R_{ft}$, R_{ft} is the riskless rate of return, $z_{1t}, \ldots , z_{qt}$ represent q conditioning variables upon which the "betas" β_{it} may depend, and ε_{it} is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \ldots , \varepsilon_{nt})' = JW_t , \ t = 1, \ldots , \ T , \tag{5}$$

where J is an unknown, non-singular matrix and the distribution of the vector $w = vec\,(W)$, $W = [W_1, \ldots , W_T]'$ is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted ν).

(a) Put the model (4) in matrix notation.

(b) On assuming that the vectors $W_1, \ldots , W_T$ are i.i.d. $N[0, I_n]$, describe the likelihood ratio test for $\mathcal{H}_E$, and discuss how this test could be implemented.

(c) Propose a procedure for testing whether the errors $W_1, \ldots , W_T$ are i.i.d. $N[0, I_n]$.

(d) If another distribution is assumed for w (such as a heavy-tailed distribution), discuss how such a test could be implemented.

15 points 5. Consider a time series of asset returns R_t, $t = 1, \ldots, T$, which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathrm{P}(S < s) = \begin{cases} -\sigma^{\alpha}|t|^{\alpha}[1 - i\beta \text{ sign}(t)\tan\frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma|t|[1 + i\beta\frac{\pi}{2} \text{ sign}(t) \ln|t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \tag{6}$$

(a) Discuss the interpretation of the different parameters μ, σ, α and β.

(b) Why are stable random variables called "stable"?

(c) On assuming that $\beta = 0$, propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \tag{7}$$

(d) On assuming that $\beta = 0$, discuss how a confidence set for α could be built.