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McGill University
ECN 706
Special topics in econometrics
Mid-term exam

Time allowed: 1.5 hour

25 points 1. Consider the following equilibrium model:

$$\begin{aligned} &Q_t = a + bp_t + u_{1t}\,, \\ &p_t = c + dp_{t-1} + u_{2t} \qquad ,\ t = 1,\, \ldots\,,\, T \\ &p_0 \text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t},\ u_{2t})'$, $t = 1, \ldots, T$ are independent $N[0,\, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1,\, \ldots\,,\, p_T)'$

(a) sequentially exogenous?
(b) exogenous?
(c) strongly exogenous?
(d) Further, does Q_t cause p_t in the sense of Granger?

Justify your answers.

25 points 2. Define the following notions:

(a) unbiased test;
(b) α-similar test;
(c) test with Neyman α-structure.

25 points 3. Consider the linear regression model

$$y = X\beta + u \tag{0.1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of unobserved error terms. Suppose the elements of u are independent and identically distributed according to a $\sigma t(1)$ distribution, where $t(1)$ represents a Student t distribution with 1 degree of freedom and σ is an unknown constant.

(a) Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such that the size of the test is exactly equal to $\alpha = 0.05$.

(b) Propose a test for detecting serial dependence between the errors $u_1, \ldots, u_T$ such the size of the test is exactly equal to $\alpha = 0.05$.

25 points 4. Consider the linear regression model

$$y = X\beta + u \tag{0.2}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of unobserved error terms, and $k \geq 3$. We wish to test the hypothesis

$$H_0 : \beta_2\beta_3 = 1. \tag{0.3}$$

(a) If $u \sim \mathrm{N}[0, \sigma^2 I_T]$, describe the likelihood ratio (LR) criterion test for testing H_0 against the unrestricted model.

(b) Propose a bound for the null distribution the above LR statistic.

(c) Can you suggest a simulation-based procedure which could eventually improve the above bound?