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McGill University
ECN 706
Special topics in econometrics
Mid-term exam

No documentation allowed
Time allowed: 2 hours

30 points 1. Provide brief answers to the following questions (maximum of 1 page per question).

(a) Explain the difference between the "level" of a test and its "size".

(b) Explain the difference between the "level" of a confidence set and its "size".

(c) Discuss the link between tests and confidence sets: how confidence sets can be derived from tests, and vice-versa.

(d) Explain what the Bahadur-Savage theorem entails for testing in nonparametric models.

(e) Suppose we wish to test the hypothesis

$$H_0: \quad X_1, \ldots, X_n \text{ are independent random variables each with a distribution symmetric about zero.} \tag{1}$$

What condition should this test satisfy to have level 0.05.

35 points 2. Consider the standard simultaneous equations model:

$$y = Y\beta + X_1\gamma + u\,, \tag{2}$$
$$Y = X_1\Pi_1 + X_2\Pi_2 + V\,, \tag{3}$$

where y and Y are $T \times 1$ and $T \times G$ matrices of endogenous variables, X_1 and X_2 are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of structural disturbances, and $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of reduced-form disturbances,

$$X = [X_1,\, X_2] \text{ is a full-column rank } T \times k \text{ matrix} \tag{4}$$

where $k = k_1 + k_2$. and

$$u \text{ and } X \text{ are independent;} \tag{5}$$
$$u \sim N\big[0,\, \sigma_u^2\, I_T\big]\,. \tag{6}$$

(a) When is the parameter β identified? Explain your answer.

(b) When is the parameter β weakly identified? Explain your answer.

(c) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0\,. \tag{7}$$

i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.

ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.

iii. Discuss the properties of the latter procedure if the model for Y is in fact

$$Y = X_1\mathit{\Pi}_1 + X_2\mathit{\Pi}_2 + X_3\mathit{\Pi}_3 + V \tag{8}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.

20 points 3. Consider the following simplified equilibrium model:

$$\begin{aligned} D_t &= \alpha + 2p_t + u_{1t}\,, \\ S_t &= c + u_{2t}\,, \\ Q_t &= D_t = S_t \qquad ,\; t = 1, \ldots, T \end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, and Q_t the quantity produced and sold. We suppose that the vectors $(u_{1t},\; u_{2t})'$, $t = 1, \ldots, T$, are independent and $N\,[0,\; I_2]$.

(a) Find the reduced form of this model.

(b) For which parameters is the vector $Q = (Q_1, \ldots, Q_T)'$ exogenous? Justify your answer.

(c) For which parameters is the vector $p = (p_1, \ldots, p_T)'$ exogenous? Justify your answer.

(d) Are the variables Q_t and p_t simultaneous?

15 points 4. Describe the main statistical problems as decision problems.

(a) Explain the difference between a *nonrandomized* decision rule and a *randomized* decision rule.

(b) Define the risk function for each one of these two types of rule.

(c) When is a decision rule *admissible*?