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McGill University
ECON 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

15 points 1. Provide brief answers to the following questions (maximum of 1 page per question).

(a) Explain the difference between the "level" of a test and its "size".

(b) Explain the difference between the "level" of a confidence set and its "size".

(c) Discuss the link between tests and confidence sets: how confidence sets can be derived from tests, and vice-versa.

(d) Explain what the Bahadur-Savage theorem entails for testing in nonparametric models.

(e) Suppose we wish to test the hypothesis

$$H_0: \quad X_1, \ldots, X_n \text{ are independent random variables each with a distribution symmetric about zero.} \tag{1}$$

What condition should this test satisfy to have level 0.05.

20 points 2. Define the following notions:

(a) Fisher information;

(b) sufficient statistic;

(c) identifiable parameter;

(d) locally identifiable parameter;

(e) identification-robust test;

(f) unbiased estimator;

(g) unbiased test;

(h) invariant test;

(i) Neyman-Pearson test;

(j) $C(\alpha)$ test.

20 points 3. Consider the equilibrium model:

$$\begin{aligned} q_t &= ap_t + b + u_t \,, \\ S_t &= \alpha p_t + \beta x_t + \nu_t \,, \\ q_t &= S_t \,, \end{aligned}$$

where q_t is the quantity demanded, p_t is the price, S_t is quantity supplied, x_t is an exogenous variable, and the vectors $(u_t, \nu_t)'$, $t = 1, \ldots, n$ are independent with the same distribution

$$N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \sigma_{u\nu} \\ \sigma_{u\nu} & \sigma_\nu^2 \end{pmatrix}\right] .$$

(a) Find the reduced form of this model.

(b) How are the parameters of this reduced form related to the structural form? Is this model underidentified, just identified, or overidentified ?

(c) Find the maximum likelihood estimators of the reduced-form coefficients.

(d) Find the maximum likelihood estimators of the structural-form coefficients.

20 points 4. Consider the linear regression model

$$y = X\beta + u \tag{2}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and u is a $T \times 1$ vector of unobserved error terms.

(a) Suppose the elements of u are independent and identically distributed according to a $N[0, \sigma^2]$ distribution, where σ^2 is an unknown constant, and $k > 1$. We wish to build a confidence interval with level 0.95 for the ratio $\theta = \beta_2^3/\beta_1$. Propose a method for doing this.

(b) Suppose the elements of u are independent and identically distributed like a σB distribution, where B follows a Bernoulli distribution on $\{-1, +1\}$, i.e.

$$\mathsf{P}[B = 1] = \mathsf{P}[B = -1] = 0.5 \,, \tag{3}$$

and σ is an unknown constant.

i. Is the least squares estimator unbiased in this model? If so, is it best linear unbiased?

ii. Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such the size of the test is exactly equal to $\alpha = 0.05$.

iii. Discuss how β and σ could be estimated by maximum likelihood.

25 points 5. Consider the standard simultaneous equations model:

$$\begin{aligned} y &= Y\beta + X_1\gamma + u\,, && (4) \\ Y &= X_1\Pi_1 + X_2\Pi_2 + V\,, && (5) \end{aligned}$$

where y and Y are $T \times 1$ and $T \times G$ matrices of endogenous variables, X_1 and X_2 are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of structural disturbances, and $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of reduced-form disturbances,

$$X = [X_1,\, X_2] \text{ is a full-column rank } T \times k \text{ matrix} \qquad (6)$$

where $k = k_1 + k_2$. and

$$u \text{ and } X \text{ are independent;} \qquad (7)$$

$$u \sim N\big[0,\, \sigma_u^2\, I_T\big]\,. \qquad (8)$$

(a) When is the parameter β identified? Explain your answer.

(b) When is the parameter β weakly identified? Explain your answer.

(c) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0\,. \qquad (9)$$

i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.

ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.

iii. Discuss the properties of the latter procedure if the model for Y is in fact

$$Y = X_1\Pi_1 + X_2\Pi_2 + X_3\Pi_3 + V \qquad (10)$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.

(d) Describe an exact identification-robust confidence set for β. Is this set bounded with probability one ? If not, give a sufficient condition that would ensure it is bounded.