Jean-Marie Dufour
December 18, 2017

McGill University
ECN 706
Special topics in econometrics
Final exam

Time allowed: to be handed on December 19, 2017
E-mail for submission: jean-marie.dufour@mcgill.ca

20 points 1. Define the following notions:

(a) sufficient statistic;
(b) ancillary statistic;
(c) Fisher information;
(d) complete statistic;
(e) identification;
(f) identification-robust test;
(g) nuisance parameter;
(h) $C(\alpha)$ statistic;
(i) unbiased test;
(j) invariant test.

15 points 2. Let (Y_i, X_i), $i = 1, \ldots, n$, be observations such that the conditional likelihood of $Y = (Y_1, \ldots, Y_n)$ given $X = (X_1, \ldots, X_n)'$ has the form:

$$L_n(\theta) = \prod_{i=1}^{n} f(y_i \mid x_i, \theta)$$

where θ is a $p \times 1$ parameter vector. Further, suppose standard regularity conditions [such as those used by Gouriéroux and Monfort (1995, Chapter 7)] are satisfied. We consider an implicit hypothesis $H_0 : g(\theta) = 0$, where $g(\theta)$ is an $r \times 1$ vector such that the matrix $\partial g / \partial \theta'$ has rank r $(1 \leq r \leq p)$.

(a) Derive the asymptotic distribution (under H_0) of the Wald statistic for testing H_0.

(b) Show that this test is consistent.

15 points 3. Explain in an intuitive way the principles underlying the following tests: Wald test, Rao score test, likelihood ratio, Neyman's $C(\alpha)$, and Hausman test.

20 points 4. Consider the linear regression model

$$y = X\beta + u \tag{1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and u is a $T \times 1$ vector of unobserved error terms.

(a) Suppose the elements of u are independent and identically distributed according to a $N[0, \sigma^2]$ distribution, where σ^2 is an unknown constant, and $k > 1$. We wish to build a confidence interval with level 0.95 for the ratio $\theta = \beta_2^3/\beta_1$. Propose a method for doing this.

(b) Suppose the elements of u are independent and identically distributed like a σB distribution, where B follows a Bernoulli distribution on $\{-1, +1\}$, i.e.

$$\mathsf{P}[B = 1] = \mathsf{P}[B = -1] = 0.5\,, \tag{2}$$

and σ is an unknown constant.

i. Is the least squares estimator unbiased in this model? If so, is it best linear unbiased?

ii. Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such the size of the test is exactly equal to $\alpha = 0.05$.

iii. Discuss how β and σ could be estimated by maximum likelihood.

30 points 5. Consider the following simultaneous equations model:

$$\begin{aligned} y &= Y\beta + X_1\gamma + u\,, & (3)\\ Y &= X_1\Pi_1 + X_2\Pi_2 + V\,, & (4) \end{aligned}$$

where y and Y are $T \times 1$ and $T \times G$ matrices of endogenous variables, X_1 and X_2 are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of random disturbances, $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of random disturbances,

$$X = [X_1,\, X_2] \text{ is a } T \times k \text{ full-column rank matrix,} \tag{5}$$

where $k = k_1 + k_2$, and

$$u \text{ and } X \text{ are independent,} \tag{6}$$

$$u \sim N\big[0,\, \sigma_u^2 I_T\big]\,. \tag{7}$$

(a) Discuss the conditions under which the parameters of equation (3) are identified.

(b) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0\,. \tag{8}$$

  i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.
  ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.
  iii. If $G = 1$, propose an exact confidence region for β;
  iv. If $G \geq 2$, propose an exact confidence region for β.

(c) Discuss how the following outcomes can be interpreted:

  i. the confidence set for β is equal to the whole real line;
  ii. the confidence set for β is empty

(d) Discuss the properties of the procedures proposed in the above sub-question if the model for Y is in fact

$$Y = X_1 \mathit{\Pi}_1 + X_2 \mathit{\Pi}_2 + X_3 \mathit{\Pi}_3 + V \tag{9}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.

(e) Describe an exact procedure for testing an hypothesis of the form:

$$H_0 : \beta = \beta_0 \text{ and } \gamma = \gamma_0 \tag{10}$$

where β_0 and γ_0 are given values.

(f) Propose an exact confidence region for γ.