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April 29, 2016

McGill University
ECON 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

20 points 1. Provide brief answers to the following questions (maximum of 1 page per question).

(a) Explain the difference between the "level" of a test and its "size".

(b) Explain the difference between the "level" of a confidence set and its "size".

(c) Discuss the link between tests and confidence sets: how confidence sets can be derived from tests, and vice-versa.

(d) Explain what the Bahadur-Savage theorem entails for testing in nonparametric models.

(e) Suppose we wish to test the hypothesis

$$H_0 : \quad \begin{array}{l} X_1, \ldots, X_n \text{ are independent random variables} \\ \text{each with a distribution symmetric about zero.} \end{array} \tag{1}$$

What condition should this test satisfy to have level 0.05.

20 points 2. Consider the following equilibrium model:

$$\begin{aligned} D_t &= a + bp_t + u_{1t}\,, \\ S_t &= c + dp_{t-1} + ex_t + fx_{t-1} + u_{2t}\,, \\ Q_t &= D_t = S_t \qquad , \; t = 1, \ldots, T \end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, Q_t the quantity produced, x_t is an exogenous variable, p_0 and x_0 are fixed, and $u_t = (u_{1t}, u_{2t})'$ is random vector such that $E(u_t) = 0$.

(a) Give the structural form associated with this model.

(b) Give the reduced form of this model.

(c) Find the short-term multipliers for p_t and Q_t.

(d) Find the final form of the model.

(e) Find the dynamic multipliers for p_t.

(f) Find the long-run form of the model and the long-term multipliers for p_t and Q_t.

20 points 3. Consider the following assumptions:

H1: the variables $Y_1, \ldots, Y_n$ are independent and follow the same distribution with density $f(y;\theta)$, $\theta \in \Theta \subseteq \mathbb{R}^p$;

H2: the interior of Θ is non-empty, and θ_0 belongs to the interior of Θ;

H3: the true unknown value θ_0 is identifiable;

H4: the log-likelihood

$$L_n(y;\theta) = \sum_{i=1}^{n} \log[f(y_i;\theta)] \text{ is continuous in } \theta\,;$$

H5: $\mathsf{E}_{\theta_0}[\log f(Y_i;\theta)]$ is finite;

H6: the log-likelihood is such that $\frac{1}{n}L_n(y;\theta)$ converges almost surely to $\mathsf{E}_{\theta_0}[\log(Y_i;\theta)]$ uniformly in $\theta \in \Theta$;

H7: the log-likelihood is twice continuously differentiable in open neighborhood of θ_0;

H8: $I_1(\theta_0) = \mathsf{E}_{\theta_0}\left[-\frac{\partial^2 \log f(Y;\theta)}{\partial\theta\,\partial\theta'}\right]$ is finite and invertible.

If $\hat{\theta}_n$ is consistent sequence of local maxima, show that the asymptotic distribution of $\sqrt{n}(\hat{\theta}_n - \theta_0)$ is $N\left[0,\, I_1(\theta_0)^{-1}\right]$.

40 points 4. Consider the following simultaneous equations model:

$$y = Y\beta + X_1\gamma + u\,, \tag{2}$$

$$Y = X_1\Pi_1 + X_2\Pi_2 + V\,, \tag{3}$$

where y and Y are $T\times 1$ and $T\times G$ matrices of endogenous variables, X_1 and X_2 are $T\times k_1$ and $T\times k_2$ matrices of exogenous variables, β and γ are $G\times 1$ and $k_1\times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1\times G$ and $k_2\times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of random disturbances, $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of random disturbances,

$$X = [X_1, X_2] \text{ is a } T \times k \text{ full-column rank matrix,} \tag{4}$$

where $k = k_1 + k_2$, and

$$u \text{ and } X \text{ are independent,} \tag{5}$$

$$u \sim N\left[0, \sigma_u^2 I_T\right]. \tag{6}$$

(a) Discuss the conditions under which the parameters of equation (2) are identified.

(b) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0 . \tag{7}$$

   i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.
   ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.
   iii. If $G = 1$, propose an exact confidence region for β;
   iv. If $G \geq 2$, propose an exact confidence region for β.

(c) Discuss how the following outcomes can be interpreted:

   i. the confidence set for β is equal to the whole real line;
   ii. the confidence set for β is empty

(d) Discuss the properties of the procedures proposed in the above sub-question if the model for Y is in fact

$$Y = X_1 \Pi_1 + X_2 \Pi_2 + X_3 \Pi_3 + V \tag{8}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.

(e) Describe an exact procedure for testing an hypothesis of the form:

$$H_0 : \beta = \beta_0 \text{ and } \gamma = \gamma_0 \tag{9}$$

where β_0 and γ_0 are given values.

(f) Propose an exact confidence region for γ.

(g) If the assumption (6) is replaced by

$$u_1, \ldots, u_T \sim \sigma\, t(2) \tag{10}$$

where $t(2)$ represents the Student t distribution with 2 degrees of freedom and σ is an unknown positive constant, propose an exact method for testing $H_0(\beta_0)$.