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McGill University
ECN 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

10 points 1. Let $\ell(Y;\theta)$ be the likelihood function for the sample $Y = (Y_1, \ldots, Y_n)'$. Show that

$$I(\theta) = E\left[-\frac{\partial^2 \log \ell(Y;\theta)}{\partial\theta\,\partial\theta'}\right].$$

20 points 2. Consider the following assumptions:

H1: the variables $Y_1, \ldots, Y_n$ are independent and follow the same distribution with density $f(y;\theta)$, $\theta \in \Theta \subseteq \mathbb{R}^p$;

H2: the interior of Θ is non-empty, and θ_0 belongs to the interior of Θ;

H3: the true unknown value θ_0 is identifiable;

H4: the log-likelihood

$$L_n(y;\theta) = \sum_{i=1}^{n} \log[f(y_i;\theta)] \text{ is continuous in } \theta;$$

H5: $\mathsf{E}_{\theta_0}[\log f(Y_i;\theta)]$ is finite;

H6: the log-likelihood is such that $\frac{1}{n}L_n(y;\theta)$ converges almost surely to $\mathsf{E}_{\theta_0}[\log(Y_i;\theta)]$ uniformly in $\theta \in \Theta$;

H7: the log-likelihood is twice continuously differentiable in open neighborhood of θ_0;

H8: $I_1(\theta_0) = \mathsf{E}_{\theta_0}\left[-\frac{\partial^2 \log f(Y;\theta)}{\partial\theta\,\partial\theta'}\right]$ is finite and invertible.

If $\hat{\theta}_n$ is consistent sequence of local maxima, show that the asymptotic distribution of $\sqrt{n}\left(\hat{\theta}_n - \theta_0\right)$ is $N\left[0,\, I_1(\theta_0)^{-1}\right]$.

20 points 3. State and prove the *Neyman-Pearson theorem*.

10 points 4. Define the following notions:

(a) unbiased test;

(b) α-similar test;

(c) test with Neyman α-structure.

10 points 5. Demonstrate the following relationship between identifiability and unbiased estimation:
if a function $g(\theta)$ of a parameter θ is not identifiable, then there is no unbiased estimator of $g(\theta)$.

30 points 6. Consider the standard simultaneous equations model:

$$y = Y\beta + X_1\gamma + u, \tag{1}$$
$$Y = X_1\Pi_1 + X_2\Pi_2 + V, \tag{2}$$

where y and Y are $T \times 1$ and $T \times G$ matrices of endogenous variables, X_1 and X_2 are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of structural disturbances, and $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of reduced-form disturbances,

$$X = [X_1, X_2] \text{ is a full-column rank } T \times k \text{ matrix} \tag{3}$$

where $k = k_1 + k_2$. and

$$u \text{ and } X \text{ are independent;} \tag{4}$$
$$u \sim N\left[0, \sigma_u^2 I_T\right]. \tag{5}$$

(a) When is the parameter β identified? Explain your answer.

(b) When is the parameter β weakly identified? Explain your answer.

(c) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0. \tag{6}$$

i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.

ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.

iii. Discuss the properties of the latter procedure if the model for Y is in fact

$$Y = X_1\Pi_1 + X_2\Pi_2 + X_3\Pi_3 + V \tag{7}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.