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McGill University
EOCN 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

20 points 1. Provide brief answers to the following questions (maximum of 1 page per question).

(a) Explain the difference between the "level" of a test and its "size".

(b) Explain the difference between the "level" of a confidence set and its "size".

(c) Discuss the link between tests and confidence sets: how confidence sets can be derived from tests, and vice-versa.

(d) Explain what the Bahadur-Savage theorem entails for testing in nonparametric models.

(e) Suppose we wish to test the hypothesis

$$H_0: \quad \begin{array}{l} X_1, \ldots, X_n \text{ are independent random variables} \\ \text{each with a distribution symmetric about zero.} \end{array} \tag{1}$$

What condition should this test satisfy to have level 0.05.

10 points 2. Let $\ell(Y;\theta)$ be the likelihood function for the sample $Y = (Y_1, \ldots, Y_n)'$. Show that

$$I(\theta) = E\left[-\frac{\partial^2 \log \ell(Y;\theta)}{\partial\theta\,\partial\theta'}\right].$$

20 points 3. Consider the following assumptions:

H1: the variables $Y_1, \ldots, Y_n$ are independent and follow the same distribution with density $f(y;\theta)$, $\theta \in \Theta \subseteq \mathbb{R}^p$;

H2: the interior of Θ is non-empty, and θ_0 belongs to the interior of Θ;

H3: the true unknown value θ_0 is identifiable;

H4: the log-likelihood

$$L_n(y;\,\theta) = \sum_{i=1}^{n} \log\left[f(y_i;\,\theta)\right] \text{ is continuous in } \theta\,;$$

H5: $\mathsf{E}_{\theta_0}\left[\log f(Y_i;\,\theta)\right]$ is finite;

H6: the log-likelihood is such that $\frac{1}{n}L_n(y;\,\theta)$ converges almost surely to $\mathsf{E}_{\theta_0}\left[\log(Y_i;\,\theta)\right]$ uniformly in $\theta \in \Theta$;

H7: the log-likelihood is twice continuously differentiable in open neighborhood of θ_0;

H8: $I_1(\theta_0) = \mathsf{E}_{\theta_0}\left[-\frac{\partial^2 \log f(Y;\,\theta)}{\partial\theta\,\partial\theta'}\right]$ is finite and invertible.

If $\hat{\theta}_n$ is consistent sequence of local maxima, show that the asymptotic distribution of $\sqrt{n}\big(\hat{\theta}_n - \theta_0\big)$ is $N\big[0,\, I_1(\theta_0)^{-1}\big]$.

20 points 4. State and prove the *Neyman-Pearson theorem*.

30 points 5. Consider the standard simultaneous equations model:

$$\begin{aligned} y &= Y\beta + X_1\gamma + u\,, && (2)\\ Y &= X_1\Pi_1 + X_2\Pi_2 + V\,, && (3) \end{aligned}$$

where y and Y are $T\times 1$ and $T\times G$ matrices of endogenous variables, X_1 and X_2 are $T\times k_1$ and $T\times k_2$ matrices of exogenous variables, β and γ are $G\times 1$ and $k_1\times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1\times G$ and $k_2\times G$ matrices of unknown coefficients, $u = (u_1,\,\ldots,\,u_T)'$ is a $T\times 1$ vector of structural disturbances, and $V = [V_1,\,\ldots,\,V_T]'$ is a $T\times G$ matrix of reduced-form disturbances,

$$X = [X_1, X_2] \text{ is a full-column rank } T\times k \text{ matrix} \qquad (4)$$

where $k = k_1 + k_2$. and

$$u \text{ and } X \text{ are independent;} \qquad (5)$$

$$u \sim N\big[0,\,\sigma_u^2 I_T\big]\,. \qquad (6)$$

(a) When is the parameter β identified? Explain your answer.

(b) When is the parameter β weakly identified? Explain your answer.

(c) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0 \,. \tag{7}$$

i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.

ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.

iii. Discuss the properties of the latter procedure if the model for Y is in fact

$$Y = X_1\Pi_1 + X_2\Pi_2 + X_3\Pi_3 + V \tag{8}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.

(d) Describe an exact identification-robust confidence set for β. Is this set bounded with probability one ? If not, give a sufficient condition that would ensure it is bounded.