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McGill University
ECN 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

30 points 1. Consider the model

$$X_t = \beta_0 + \sum_{k=1}^{p} \lambda_k X_{t-k} + u_t \, , \; t = 1 \, , \, \ldots \, , \, n \tag{1}$$

and the problem of testing the hypothesis

$$H_0 : \sum_{k=1}^{p} \lambda_k = 1 \tag{2}$$

in the context of model (1).

(a) If $u_t \overset{i.i.d.}{\sim} N[0\,,\,\sigma^2]$ and p is known, propose an exact method for testing H_0.

(b) If $u_t \overset{i.i.d.}{\sim} \sigma t(1)$ and p is known, propose an exact method for testing H_0.
[$t(1)$ represents a Student t variable with 1 degree of freedom.]

(c) Discuss the problem of testing H_0 when p is unknown.

40 points 2. Consider the following simultaneous equations model:

$$\begin{aligned} y &= Y\beta + X_1\gamma + u \, , & (3) \\ Y &= X_1 \Pi_1 + X_2 \Pi_2 + V \, , & (4) \end{aligned}$$

where y and Y are $T \times 1$ and $T \times G$ matrices of endogenous variables, X_1 and X_2 are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, β and γ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of random disturbances, $V = [V_1, \ldots, V_T]'$ is a $T \times G$ matrix of random disturbances,

$$X = [X_1,\, X_2] \text{ is a } T \times k \text{ full-column rank matrix,} \tag{5}$$

where $k = k_1 + k_2$, and

$$u \text{ and } X \text{ are independent,} \tag{6}$$
$$u \sim N\big[0,\, \sigma_u^2\, I_T\big]\,. \tag{7}$$

(a) Discuss the conditions under which the parameters of equation (3) are identified;

(b) if $G = 1$, propose an exact confidence region for β;

(c) if $G \geq 2$, propose an exact confidence region for β;

(d) if $G \geq 2$, propose an exact confidence region for each component of β;

(e) describe an exact procedure for testing an hypothesis of the form:

$$H_0 : \beta = \beta_0 \text{ and } \gamma = \gamma_0 \tag{8}$$

where β_0 and γ_0 are given values;

(f) propose an exact confidence region for γ.

30 points 3. Consider the process described by the following model:

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \begin{bmatrix} 1 - 0.5B & 0 \\ -0.5B & 1 - 0.2B \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \tag{9}$$

où $t \in \mathbb{Z}$, $a_t = [a_{1t},\, a_{2t}]'$ is a sequence of *i.i.d.* $N[0,\, \Sigma]$ random vectors with

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{10}$$

(a) What is the type of this process?

(b) Is this process strictly stationary? Why?

(c) Does this process have a Wold representation? If so, give it.

(d) Is this process invertible? Why?

(e) Does this process has an autoregressive representation? If so, give it.

(f) Does the variable X_{2t} cause X_{1t} in the sense of Granger ? Justify your answer.

(g) Does the variable X_{1t} cause X_{2t} in the sense of Granger ? Justify your answer.

(h) Does the variable X_{2t} cause X_{1t} at all horizons ? Justify your answer.

(i) Does the variable X_{1t} cause X_{2t} at all horizons ? Justify your answer.