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McGill University
ECON 257D
Honours Statistics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

30 points 1. Respond by TRUE, FALSE or UNCERTAIN to each one of the following statements and explain your answer (Maximum: 1 page per statement)

(a) If a random variable X has zero variance, it is unpredictable.

(b) If the correlation between two random variables X and Y is one, this means that one of them has variance zero.

(c) If X is a random variable with zero variance, then its covariance with any other random variable Y cannot be negative.

(d) If X is a random variable with variance equal to zero, then its correlation with any other random variable Y must be positive.

(e) All the elements of a covariance matrix must be positive.

25 points 2. Describe the classical linear model.

25 points 3. State and prove the Gauss-Markov theorem.

20 points 4. Suppose that

$$y = X\beta + \varepsilon$$

satisfies the assumptions of the classical linear model [where X is $T \times k$ matrix], and consider a vector y_0 on m additional periods (or units) satisfying

$$y_0 = X_0\beta + \varepsilon_0 \,,$$

$$\mathsf{E}\,(\varepsilon_0) = 0 \,, \mathsf{V}\,(\varepsilon_0) = \sigma^2 I_m \,, \mathsf{E}\,(\varepsilon\varepsilon_0') = 0 \,,$$

where X_0 is a known $m \times k$ matrix, but y_0 and ε_0 are not observed. We wish to predict y_0 by using y and X_0. Let $\hat{\beta}$ is the ordinary least squares estimator of β based on y.

(a) Show that $\hat{y}_0 = X_0\hat{\beta}$ is an unbiased predictor of y_0.

(b) Derive the covariance matrices of $\hat{y}_0$ and $y_0 - \hat{y}_0$.

(c) What can you say about the optimality of $\hat{y}_0$ as a predictor of y_0?