

McGill University
ECON 257D
Honours Statistics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

- 30 points 1. Respond by TRUE, FALSE or UNCERTAIN to each one of the following statements and explain your answer (Maximum: 1 page per statement)
- (a) If a random variable X has zero variance, it is unpredictable.
 - (b) If the correlation between two random variables X and Y is one, this means that one of them has variance zero.
 - (c) If X is a random variable with zero variance, then its covariance with any other random variable Y cannot be negative.
 - (d) If X is a random variable with variance equal to zero, then its correlation with any other random variable Y must be positive.
 - (e) All the elements of a covariance matrix must be positive.
- 25 points 2. Describe the classical linear model.
- 25 points 3. State and prove the Gauss-Markov theorem.
- 20 points 4. Suppose that

$$y = X\beta + \varepsilon$$

satisfies the assumptions of the classical linear model [where X is $T \times k$ matrix], and consider a vector y_0 on m additional periods (or units) satisfying

$$y_0 = X_0\beta + \varepsilon_0,$$

$$\mathbf{E}(\varepsilon_0) = 0, \mathbf{V}(\varepsilon_0) = \sigma^2 I_m, \mathbf{E}(\varepsilon\varepsilon_0') = 0,$$

where X_0 is a known $m \times k$ matrix, but y_0 and ε_0 are not observed. We wish to predict y_0 by using y and X_0 . Let $\hat{\beta}$ is the ordinary least squares estimator of β based on y .

- (a) Show that $\hat{y}_0 = X_0\hat{\beta}$ is an unbiased predictor of y_0 .
- (b) Derive the covariance matrices of \hat{y}_0 and $y_0 - \hat{y}_0$.
- (c) What can you say about the optimality of \hat{y}_0 as a predictor of y_0 ?