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McGill University
ECON 257D
Honours Statistics
Final exam

No documentation allowed
Time allowed: 3 hours

20 points 1. Suppose that

$$y = X\beta + \varepsilon \tag{1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ matrix of observations on explanatory variables, $\beta = (\beta_1, \ldots, \beta_k)'$ is a $k \times 1$ vector of fixed parameters, and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_T)'$ is a $T \times 1$ vector of random disturbances.

(a) Describe which extra assumptions are needed for y to satisfy the classical linear model.

(b) Suppose now we know the matrix X_0 of explanatory variables for m additional periods (or observations). We wish to predict the corresponding values of y:

$$y_0 = X_0\beta + \varepsilon_0$$

where

$$\mathsf{E}(\varepsilon_0) = 0\ , \mathsf{V}(\varepsilon_0) = \sigma^2 I_m\ , \mathsf{E}(\varepsilon\varepsilon_0') = 0\ .$$

Propose a linear unbiased predictor $\hat{y}_0$ of y_0, and show that it is indeed unbiased.

(c) Derive the covariance matrix of $\hat{y}_0$.

(d) Show that $\hat{y}_0$ is best linear unbiased.

40 points 2. Under the assumptions of the classical linear model on (1), suppose that the elements of $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_T)'$ are $i.i.d$ according to a $\mathrm{N}[0, \sigma^2]$. Propose:

(a) a confidence interval with level 0.95 for the first component of β_1 (the first element of β);

(b) a confidence interval with level 0.95 for the error variance σ^2;

(c) a test for the hypothesis that $H_0 : \beta_1 = 0$;

(d) a test for the hypothesis that $H_0 : \beta_1 = 1$;

(e) a test for the hypothesis that $H_0 : \beta_1 = \beta_2 = 0$.

20 points 3. In the context of the classical linear regression model (with an intercept), answer the following quastions:

(a) define R^2 and $\bar{R}^2$;

(b) show that $\overline{R}^2 \leq R^2 \leq 1$;

(c) give conditions under which $\overline{R}^2 = R^2$;

(d) can R^2 be negative ? If so, when?

(e) can $\bar{R}^2$ be negative ? If so, when?

20 points 4. In the context of the classical linear regression model,

$$y = X\beta + \varepsilon \quad , \quad \varepsilon \sim N\left[0, \sigma^2 I_T\right] \tag{2}$$

$$y : T \times 1, \quad X : T \times k\,, \quad \varepsilon : T \times 1 \tag{3}$$

we wish to analyze whether the least squares residuals

$$\hat{\varepsilon} = y - X\hat{\beta} \tag{4}$$

behave as expected under the assumption that the model is correctly specified.

(a) Establish the mean and covariance matrix of $\hat{\varepsilon}$.

(b) What is the distribution of $\hat{\varepsilon}$?

(c) Do the elements of $\hat{\varepsilon}$ have the same variance ? If not, propose a method for making all these variances equal.

(d) Are the elements of $\hat{\varepsilon}$ uncorrelated ?

(e) Propose a method for deciding whether a given residual is surprisingly "large".

(f) Propose a method for deciding whether the residuals of the model contain an "outlier" ?

(g) Propose a method for testing the errors are "homoskedastic" against an alternative where the variance is increasing with the observation index $(t = 1, \ldots, T)$.