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# ECON 257
# EXERCISES 7

## Classical linear model: Analysis of residuals
## Review questions

In the context of the classical linear regression model,

$$y = X\beta + \varepsilon \quad , \quad \varepsilon \sim N\left[0, \sigma^2 I_T\right] \tag{1}$$

$$y : T \times 1, \quad X : T \times k, \quad \varepsilon : T \times 1 \tag{2}$$

we wish to analyze whether the least squares residuals

$$\hat{\varepsilon} = y - X\hat{\beta} \tag{3}$$

behave as expected under the assumption that the model is correctly specified.

1. Establish the mean and covariance matrix of $\hat{\varepsilon}$.
2. What is the distribution of $\hat{\varepsilon}$ ?
3. Do the elements of $\hat{\varepsilon}$ have the same variance ? If not, propose a method for making all these variances equal.
4. Are the elements of $\hat{\varepsilon}$ uncorrelated ?
5. Propose a method for deciding whether a given residual is surprisingly "large".
6. Propose a method for deciding whether the residuals of the model contain an "outlier" ?
7. Propose a method for testing the errors are "homoskedastic" against an alternative where the variance is increasing with the observation index $(t = 1, \ldots, T)$.