

**ECON 257**  
**EXERCISES 5**

**Classical linear model: Specification errors**  
**Review questions**

Consider a  $T \times 1$  vector of observations  $y$  such that

$$y = X\beta + \varepsilon \quad (1)$$

where all the assumptions the classical linear model are satisfied.

1. Suppose that

$$y = X\beta + \varepsilon = X_1\beta_1 + x_k\beta_k + \varepsilon \quad (2)$$

where  $X_1$  is a  $T \times (k-1)$  fixed matrix and  $x_k$  is a  $T \times 1$  fixed vector. Instead of (2), we estimate the incomplete model:

$$y = X_1\gamma + \varepsilon. \quad (3)$$

Discuss the properties of the least squares estimator  $\hat{\gamma}$  based on the incomplete model (3) as an estimator of  $\beta_1$ .

- (a) Find the expected value of  $\hat{\gamma}$ .
  - (b) Is  $\hat{\gamma}$  an unbiased estimator of  $\beta_1$ ?
  - (c) If  $\hat{\gamma}$  is not an unbiased estimator of  $\beta_1$ , give a condition under which it would be unbiased.
  - (d) Find the covariance matrix of  $\hat{\gamma}$ . Is it larger or smaller than the covariance matrix of  $\hat{\beta}_1$  based on the complete model (2)?
  - (e) Discuss the properties of the estimator of  $\sigma^2$  based on the incomplete model (3).
2. Discuss the consequence of having

$$E(\varepsilon) = \xi \neq 0 \quad (4)$$

on the properties of the least squares estimator of  $\beta$  in (1).

3. In model (1), suppose that

$$E(\varepsilon\varepsilon') = \Omega. \quad (5)$$

where  $\Omega$  is known positive definite matrix.

- (a) Is the least squares estimator  $\hat{\beta}$  unbiased ?
- (b) Find the covariance matrix  $V(\hat{\beta})$ .
- (c) Discuss the consequences of the above observations on testing the hypothesis  $\beta_1 = 0$ .
- (d) Propose a best linear unbiased estimator of  $\beta$ .