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ECON 257
EXERCISES 3

Classical linear model
Review questions

1. Suppose that

$$y = X\beta + \varepsilon \tag{1}$$

where y is a $T \times 1$ vector of observations on a dependent variable ,X is a $T \times k$ matrix of observations on explanatory variables, $\beta = (\beta_1, \ldots, \beta_k)'$ is a $k \times 1$ vector of fixed parameters, and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_T)'$ is a $T \times 1$ vector of random disturbances.

   (a) Describe which extra assumptions are needed for y to satisfy the classical linear model.

   (b) Suppose now we know the matrix X_0 of explanatory variables for m additional periods (or observations). We wish to predict the corresponding values of y:

$$y_0 = X_0\beta + \varepsilon_0$$

   where

$$\mathsf{E}(\varepsilon_0) = 0\,, \mathsf{V}(\varepsilon_0) = \sigma^2 I_m\,, \mathsf{E}\left(\varepsilon\varepsilon_0'\right) = 0\,.$$

   Propose a linear unbiased predictor $\hat{y}_0$ of y_0, and show that it is indeed unbiased.

   (c) Derive the covariance matrix of $\hat{y}_0$.

   (d) Show that $\hat{y}_0$ is best linear unbiased.

2. Describe conditions under which the least squares estimator of β can be interpreted as a maximum likelihood estimator.

3. Under the assumptions of the classical linear model on (1), suppose that the elements of $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_T)'$ are *i.i.d* according to a $\mathrm{N}[0, \sigma^2]$. Propose:

   (a) a confidence interval with level 0.95 for the first component of β_1 (the first element of β);

   (b) a confidence interval with level 0.95 for the error variance σ^2;

   (c) a test for the hypothesis that $H_0 : \beta_1 = 0$;

   (d) a test for the hypothesis that $H_0 : \beta_1 = 1$;

   (e) a test for the hypothesis that $H_0 : \beta_1 = \beta_2 = 0$.