Jean-Marie Dufour
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ECONOMETRICS 1
EXERCISES 1

Hypothesis testing I

1. Respond by TRUE, FALSE or UNCERTAIN to each one of the following statements and explain your answer (Maximum: 1 page per statement)

   (a) The level of a test is the probability of rejecting the null hypothesis when it is false.

   (b) The power of a test is the probability of rejecting the null hypothesis when it is true.

   (c) If a test has level 0.05, it also has level 0.10 .

   (d) The probability of Type I error of a test is equal to 1-power.

   (e) The probability of Type II error of a test is equal to its power.

2. A supplier sends chips whose power usage X follows a normal distribution with mean μ and variance σ^2 , i.e. $X \sim N[\mu, \sigma^2]$. We have n i.i.d. observations $X_1, \ldots, X_n$ on X. The norm for the chips is that we should have $\mu = 90$. The sample size is $n = 25$.

   (a) Propose an unbiased estimator for μ .

   (b) Propose an unbiased estimator for σ^2 .

   (c) Suppose that $\bar{X} = \sum_{i=1}^{n} X_i/n$ is equal to 105 and we know that the true variance of X is $\sigma^2 = 3600$. Test the following hypotheses at level $\alpha = 0.05$:

      i. $H_0 : \mu = 90$ against $H_1 : \mu = 100$;
      ii. $H_0 : \mu = 90$ against $H_1 : \mu > 90$;
      iii. $H_0 : \mu = 90$ against $H_1 : \mu = 80$;
      iv. $H_0 : \mu = 90$ against $H_1 : \mu < 90$.
      v. $H_0 : \mu = 90$ against $H_1 : \mu \neq 90$.

   (d) Suppose that $\bar{X} = \sum_{i=1}^{n} X_i/n$ is equal to 105 and we know that the true variance of X is $\sigma^2 = 1$. Test the following hypotheses at level $\alpha = 0.05$:

      i. $H_0 : \mu = 90$ against $H_1 : \mu = 100$;
      ii. $H_0 : \mu = 90$ against $H_1 : \mu > 90$;
      iii. $H_0 : \mu = 90$ against $H_1 : \mu = 80$;
      iv. $H_0 : \mu = 90$ against $H_1 : \mu < 90$;

v. $H_0 : \mu = 90$ against $H_1 : \mu \neq 90$.

(e) Suppose that $\bar{X} = \sum_{i=1}^{n} X_i/n$ is equal to 105 but the true variance of X is unknown. However, we know that $s^2 = 4$. Test the following hypotheses at level $\alpha = 0.05$:

i. $H_0 : \mu = 90$ against $H_1 : \mu = 100$;

ii. $H_0 : \mu = 90$ against $H_1 : \mu > 90$;

iii. $H_0 : \mu = 90$ against $H_1 : \mu = 80$;

iv. $H_0 : \mu = 90$ against $H_1 : \mu < 90$;

v. $H_0 : \mu = 90$ against $H_1 : \mu \neq 90$.