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ECON 257
EXERCISES 5

Classical linear model: Specification errors
Review questions

Consider a $T \times 1$ vector of observations y such that

$$y = X\beta + \varepsilon \tag{1}$$

where all the assumptions the classical linear model are satisfied.

1. Suppose that
$$y = X\beta + \varepsilon = X_1\beta_1 + x_k\beta_k + \varepsilon \tag{2}$$
where X_1 is a $T \times (k-1)$ fixed matrix and x_k is a $T \times 1$ fixed vector. Instead of (2), we estimate the incomplete model:
$$y = X_1\gamma + \varepsilon\,. \tag{3}$$
Discuss the properties of the least squares estimator $\hat{\gamma}$ based on the incomplete model (3) as an estimator of β_1.

   (a) Find the expected value of $\hat{\gamma}$.
   (b) Is $\hat{\gamma}$ an unbiased estimator of β_1?
   (c) If $\hat{\gamma}$ is not an unbiased estimator of β_1, give a condition under which it would be unbiased.
   (d) Find the covariance matrix of $\hat{\gamma}$. Is it larger or smaller than the covariance matrix of $\hat{\beta}_1$ based on the complete model (2)?
   (e) Discuss the properties of the estimator of σ^2 based on the incomplete model (3).

2. Discuss the consequence of having
$$\mathsf{E}(\varepsilon) = \xi \neq 0 \tag{4}$$
on the properties of the least squares estimator of β in (1).

3. In model (1), suppose that
$$\mathsf{E}\big(\varepsilon\varepsilon'\big) = \Omega\,. \tag{5}$$
where Ω is known positive definite matrix.

(a) Is the least squares estimator $\hat{\beta}$ unbiased ?

(b) Find the covariance matrix $\mathsf{V}(\hat{\beta})$.

(c) Discuss the consequences of the above observations on testing the hypothesis $\beta_1 = 0$.

(d) Propose a best linear unbiased estimator of β.