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**McGill University**
**ECON 763**
**Financial econometrics**
**Mid-term exam**

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

(a) If a random variable has finite second moments, it has finite moments at all higher orders.

(b) Any stationary process of order 5 is also stationary of order 2.

(c) Any strictly stationary process is in $L_2$.

(d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.

(e) Non-invertible moving processes have no covariance generating function.

20 points 2. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$ , $\forall k \in \mathbb{Z}$ ;

(b) $|\gamma(k)| \leq \gamma(0)$ , $\forall k \in \mathbb{Z}$ ;

(c) the function $\gamma(k)$ is positive semi-definite.

60 points 3. Consider the following models:

$$X_t = 10 + u_t - 0.75\, u_{t-1} + 0.125\, u_{t-2}, \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;

ii. $\gamma(k)$, $k = 1, \ldots, 8$;

iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of $u_t$, $u_{t-1}$, $u_{t-2}$, $u_{t-3}$ and $u_{t-4}$ in the moving average representation of $X_t$.

(f) Find the autocovariance generating function of $X_t$.

(g) Find and graph the spectral density of $X_t$.

(h) Compute the first two partial autocorrelations of $X_t$.

(i) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{10}$, $X_{11}$, $X_{12}$ and $X_{13}$ based on $X_{10}$ (only)? If so, compute these.

(j) If $X_{10} = 1, u_{10} = 2, u_9 = 1, u_8 = 0.99, u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{11}$, $X_{12}$ and $X_{13}$ based on the history of the process up to $X_{10}$? If so, compute these.