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March 3, 2010

McGill University
Econ 469: Econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Consider a $MA(1)$ model:

$$X_t = \overline{\mu} + u_t - \theta u_{t-1}\,,\ t \in \mathbb{Z}$$

where u_t is a white noise process with mean zero and variance $\sigma^2 > 0$.

(a) Prove that the first autocorrelation of this model cannot be greater than 0.5 in absolute value.
(b) Find the values of the model parameters for which this upper bound is attained.

50 points 2. Consider the following models:

$$X_t = 10 + u_t - 0.75\, u_{t-1} + 0.125\, u_{t-2} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0, 1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?
(b) Is this model invertible? Why?
(c) Compute:
   i. $E(X_t)$;
   ii. $\gamma(k)\,,\ k = 1, \ldots, 8$;
   iii. $\rho(k)\,,\ k = 1, 2, \ldots, 8$.
(d) Graph $\rho(k)\,,\ k = 1, 2, \ldots,\ 8$.
(e) Find the coefficients of $u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3}$ and u_{t-4} in the moving average representation of X_t.

(f) Compute the first two partial autocorrelations of X_t.

(g) Suppose you have the following information: $X_8 = 11$.

   i. Compute the best forecast of X_9 (in the mean square error sense) based on the information you have.

   ii. Compute the best linear forecast of X_9 (in the mean square error sense) based on the information you have.

(h) Suppose you have the following information: $X_8 = 11$, $X_9 = 10.5$ and $X_{10} = 12$.

   i. Can you compute the best forecast of X_{15} (in the mean square error sense) based on the whole history of the process up to X_{10}? If yes, provide the answer. Explain your answer.

   ii. Can you compute the best linear forecast of X_{15} (in the mean square error sense) based on the whole history of the process up to X_{10}? If yes, provide the answer. Explain your answer.

30 points 3. Let $X_1,\ X_2,\ \dots\ ,\ X_T$ be a time series.

(a) Define:

   i. the sample autocorrelations for this series;

   ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

   i. under the hypothesis that $X_1,\ X_2,\ \dots\ ,\ X_T$ are independent and identically distributed (i.i.d.);

   ii. under the hypothesis that the process follows a moving average of finite order.

(c) Describe how you would identify the process described in equation (1) in question 2.