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McGill University
ECN 467
Econ 467D2: Econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

30 points   1.   Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

    (a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;

    (b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;

    (c) the function $\gamma(k)$ is positive semi-definite.

40 points   2.   Consider the following models:

$$X_t = 10 + 0.7\, X_{t-1} - 0.2\, X_{t-2} + u_t \tag{0.1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. For each one of these models, answer the following questions.

    (a) Is this model stationary? Why?

    (b) Is this model invertible? Why?

    (c) Compute:

      i. $E(X_t)$;

      ii. $\gamma(k)$, $k = 1, \ldots, 8$;

      iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

    (d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

    (e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Compute the first four partial autocorrelations of X_t.

30 points   3.   Let X_1, X_2, $\ldots$, X_T be a time series.

    (a) Define:

        i. the sample autocorrelations for this series;

        ii. the partial autocorrelations for this series.

    (b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

        i. under the hypothesis that X_1, X_2, $\ldots$, X_T are independent and identically distributed (i.i.d.);

        ii. under the hypothesis that the process follows a moving average of finite order.

    (c) Describe how you would identify the process described in equation (0.1) in question 2.