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Time series and financial econometrics

Stable distributions in finance

1. Consider a time series of asset returns R_t, $t = 1, \ldots, T$, which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathrm{P}(S < s) = \begin{cases} -\sigma^{\alpha}|t|^{\alpha}[1 - i\beta \,\mathrm{sign}(t) \tan\frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma|t|[1 + i\beta\frac{\pi}{2} \,\mathrm{sign}(t) \ln|t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \tag{1}$$

   (a) Discuss the interpretation of the different parameters μ, σ, α and β.

   (b) Why are stable random variables called "stable"?

   (c) On assuming that $\beta = 0$, propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \tag{2}$$

   (d) On assuming that $\beta = 0$, discuss how a confidence set for α could be built.

   References: Dufour and Kurz-Kim (2010).

References

DUFOUR, J.-M., AND J.-R. KURZ-KIM (2010): "Exact Inference and Optimal Invariant Estimation for the Stability Parameter of Symmetric α-stable Distributions," *Journal of Empirical Finance*, 17(2), 180–194.