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Time series and financial econometrics

Multivariate linear linear regression and CAPM

Exercises

1. Let R_{it}, $i = 1, \ldots , n$, be returns on n securities for period t, and $\tilde{R}_{\mathrm{M}t}$ the return on a benchmark portfolio ($t = 1, \ldots , T$). The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\mathcal{H}_E : a_i = 0 \,, \quad i = 1, \ldots , n, \tag{1}$$

in the context of the MLR model

$$r_{it} = a_i + \beta_i \tilde{r}_{\mathrm{Mt}} + \varepsilon_{it} \,, \quad t = 1, \ldots , T, \; i = 1, \ldots , n, \tag{2}$$

where $r_{it} = R_{it} - R_{ft}$, $\tilde{r}_{\mathrm{Mt}} = \tilde{R}_{\mathrm{M}t} - R_{ft}$, R_{ft} is the riskless rate of return and ε_{it} is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \ldots , \varepsilon_{nt})' = JW_t \,, \; t = 1, \ldots , \; T \,, \tag{3}$$

where J is an unknown, non-singular matrix and the distribution of the vector $w = \mathrm{vec}(W)$, $W = [W_1, \ldots , W_T]'$ is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted ν).

   (a) Put the model (2) in matrix notation.
   (b) On assuming that the vectors $W_1, \ldots , W_T$ are i.i.d. $N[0, I_n]$, describe the likelihood ratio test for $\mathcal{H}_E$, and discuss how this test could be implemented.
   (c) Propose a procedure for testing whether the errors $W_1, \ldots , W_T$ are i.i.d. $N[0, I_n]$.
   (d) If another distribution is assumed for w (such as a heavy-tailed distribution), discuss how such a test could be implemented.

   Reference: Dufour and Khalaf (2002), Beaulieu, Dufour, and Khalaf (2007).

References

BEAULIEU, M.-C., J.-M. DUFOUR, AND L. KHALAF (2007): "Multivariate Tests of Mean-Variance Efficiency with Possibly Non-Gaussian Errors: An Exact Simulation-Based Approach," *Journal of Business and Economic Statistics*, 25(4), 398–410.

DUFOUR, J.-M., AND L. KHALAF (2002): "Simulation Based Finite and Large Sample Tests in Multivariate Regressions," *Journal of Econometrics*, 111(2), 303–322.