Economics 275: Time Series Econometrics **Professor: Jean-Marie Dufour**
Spring 1999

Final Exam
to be handed in before Thursday, June 3. 1999 (10:00 AM)

(35 points) 1. Consider the following model:

$$X_t = 0.5\, X_{t-1} + u_t - 0.25\, u_{t-1}$$

where $\{u_t : t \in \mathbf{Z}\}$ is an i.i.d. N(0.1) sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?
(b) Is this model invertible? Why?
(c) Compute:

1. $E(X_t)$;
2. $\gamma(k)$, $k = 1, 2, ..., 8$;
3. $\rho(k)$, $k = 1, 2, ..., 8$.

(d) Graph $\rho(k)$, $k = 1, 2, ..., 8$.
(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.
(f) Find the autocovariance generating function of X_t.
(g) Find and graph the spectral density of X_t.
(h) Compute the first four partial autocorrelation of X_t.

(10 points) 2. Let $(X_t : t \in \mathbf{Z})$ be a second-order stationary ARMA process, with equation $\phi(B)X_t = \theta(B)a_t$ where a_t is a white noise, and $\phi(z) \neq 0$ for $|z| \neq 1$. Let $\gamma(k)$ be its autocovariance function.

(a) Show there are constants $c > 0$ and s, where $0 < s < 1$, such that $|\gamma(k)| \leq C\, s^{|k|}$, $k \in \mathbf{Z}$.

(b) Show that $\sum_{k=-\infty}^{\infty} |\gamma(k)| < \infty$.

(20 points) 3. Consider again a process which follows the model:

$$X_t = 0.5\, X_{t-1} + u_t - 0.25\, u_{t-1}$$

where $\{u_t : t \in \mathbf{Z}\}$.

Suppose that for a given realization of this process, the following values have been observed:

$$X_1 = 0.644\ ,\ X_2 = 0.442\ ,\ X_3 = 0.919$$

$$X_4 = -1.573\ ,\ X_5 = 0.852\ ,\ X_6 = -0.907\ .$$

(a) Compute the best linear forecasts of X_7 and X_8 based on $X_1, X_2, \ldots, X_6$.

(b) Calculate 95% predictive confidence intervals for X_7 and X_8.

(c) If it is further known that $X_t = 0$ for $t \le 0$, compute the best linear forecast of X_{t+h}, for $h = 1, \ldots 10$, based on all the observations $\{X_t : t \le 6\}$, and graph them.

(20 points) 4. Consider the following autocorrelations which are obtained from a time series $X_1, \ldots, X_T$ of length 100 ($T = 100$):

k	1	2	3	4	5	6
r_k	0.74	0.58	0.47	0.39	0.34	0.33

We wish to test the null hypothesis that the observations are independent and identically distributed (randomness).

(a) Describe how this null hypothesis could be tested using each individual autocorrelation and a large-sample approximation for the distribution of the autocorrelations. Discuss the assumptions required for these large sample tests to be valid. Apply the procedure to $r_1, \ldots, r_6$ as given in the above table.

(b) Propose a procedure that would combine the six autocorrelations given above. Apply the procedure to $r_1, \ldots, r_6$ as given in the above table.

(c) Describe finite sample bounds tests based on autocorrelations that would allow one to test the hypothesis of randomness. Discuss the assumptions made and the problems associated with the use of a bound. Apply the procedure to $r_1, \ldots, r_6$ as given in the above table.

(d) Discuss how the method of Monte Carlo tests could be applied to obtain a finite sample test of randomness without using a bound.

(15 points) 5. Describe the Tiao-Box approach for

(a) specifying (identifying),
(b) estimating,
(c) validating,

multivariate ARMA models. [Maximum: 3 pages].