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Journal of Econometrics 132 (2006) 337–362

JOURNAL OF Econometrics

www.elsevier.com/locate/jeconom

Short run and long run causality in time series: inference

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Available online 18 April 2005

Abstract

We propose methods for testing hypothesis of non-causality at various horizons, as defined in Dufour and Renault (Econometrica 66, (1998) 1099–1125). We study in detail the case of VAR models and we propose linear methods based on running vector autoregressions at different horizons. While the hypotheses considered are nonlinear, the proposed methods only require linear regression techniques as well as standard Gaussian asymptotic distributional theory. Bootstrap procedures are also considered. For the case of integrated processes, we propose extended regression methods that avoid nonstandard asymptotics. The methods are applied to a VAR model of the US economy.

JEL classification: C1; C12; C15; C32; C51; C53; E3; E4; E52

Keywords: Granger causality; Indirect causality; Vector autoregression; Bootsrap; Macroeconomics

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doi:10.1016/j.jeconom.2005.02.003

1. Introduction

The concept of causality introduced by (Wiener, 1956) and (Granger, 1969) is now a basic notion for studying dynamic relationships between time series. The literature on this topic is considerable; see, for example, the reviews of Pierce and Haugh (1977), Newbold (1982), Geweke (1984), Lütkepohl (1991) and Gouriéroux and Monfort (1997, Chapter 10). The original definition of Granger (1969), which is used or adapted by most authors on this topic, refers to the predictability of a variable $X(t)$, where t is an integer, from its own past, the one of another variable $Y(t)$ and possibly a vector $Z(t)$ of auxiliary variables, *one period ahead*: more precisely, we say that Y causes X in the sense of Granger if the observation of Y up to time t $(Y(\tau):\tau\leqslant t)$ can help one to predict $X(t+1)$ when the corresponding observations on X and Z are available $(X(\tau), Z(\tau):\tau\leqslant t)$; a more formal definition will be given below.

Recently, however (Lütkepohl, 1993; Dufour and Renault, 1998) have noted that, for multivariate models where a vector of auxiliary variables Z is used in addition to the variables of interest X and Y, it is possible that Y does not cause X in this sense, but can still help to predict X *several periods ahead*; on this issue, see also Sims (1980), Renault et al. (1998), Giles (2002). For example, the values $Y(\tau)$ up to time t may help to predict $X(t+2)$, even though they are useless to predict $X(t+1)$. This is due to the fact that Y may help to predict Z one period ahead, which in turn has an effect on X at a subsequent period. It is clear that studying such indirect effects can have a great interest for analyzing the relationships between time series. In particular, one can distinguish in this way properties of "short-run (non-) causality" and "long-run (non-)causality".

In this paper, we study the problem of testing non-causality at various horizons as defined in Dufour and Renault (1998) for finite-order vector autoregressive (VAR) models. In such models, the non-causality restriction at horizon one takes the form of relatively simple zero restrictions on the coefficients of the VAR [see Boudjellaba et al. (1992), Dufour and Renault (1998)]. However non-causality restrictions at higher horizons (greater than or equal to 2) are generally nonlinear, taking the form of zero restrictions on multilinear forms in the coefficients of the VAR. When applying standard test statistics such as Wald-type test criteria, such forms can easily lead to asymptotically singular covariance matrices, so that standard asymptotic theory would not apply to such statistics. Further, calculation of the relevant covariance matrices—which involve the derivatives of potentially large numbers of restrictions—can become quite awkward.

Consequently, we propose simple tests for non-causality restrictions at various horizons [as defined in Dufour and Renault (1998)] which can be implemented only through linear regression methods and do not involve the use of artificial simulations [e.g., as in Lütkepohl and Burda (1997)]. This will be done, in particular, by considering multiple horizon vector autoregressions [called (p, h)-*autoregressions*] where the parameters of interest can be estimated by linear methods. Restrictions of non-causality at different horizons may then be tested through simple Wald-type (or Fisher-type) criteria after taking into account the fact that such autoregressions involve autocorrelated errors [following simple moving average processes] which are orthogonal to the regressors. The correction for the presence of autocorrelation in the errors may then be performed by using an autocorrelation consistent [or heteroskedasticity-autocorrelation-consistent (HAC)] covariance matrix estimator. Further, we distinguish between the case where the VAR process considered is stable (i.e., the roots of the determinant of the associated AR polynomial are all outside the unit circle) and the one where the process may be integrated of an unknown order (although not explosive). In the first case, the test statistics follow standard chi-square distributions while, in the second case, they may follow nonstandard asymptotic distributions involving nuisance parameters, as already observed by several authors for the case of causality tests at horizon one [see Sims et al. (1990), Toda and Phillips (1993, 1994), Toda and Yamamoto (1995), Dolado and Lütkepohl (1996), Yamada and Toda (1998)]. To meet the objective of producing simple procedures that can be implemented by least squares methods, we propose to deal with such problems by using an extension to the case of multiple horizon autoregressions of the lag extension technique suggested by Choi (1993) for inference on univariate autoregressive models and by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996) for inference on standard VAR models. This extension will allow us to use standard asymptotic theory in order to test non-causality at different horizons without making assumption on the presence of unit roots and cointegrating relations. Finally, to alleviate the problems of finite-sample unreliability of asymptotic approximations in VAR models (on both stationary and nonstationary series), we propose the use of bootstrap methods to implement the proposed test statistics.

In Section 2, we describe the model considered and introduce the notion of autoregression at horizon h [or (p, h)-autoregression] which will be the basis of our method. In Section 3, we study the estimation of (p, h)-autoregressions and the asymptotic distribution of the relevant estimators for stable VAR processes. In Section 4, we study the testing of non-causality at various horizons for stationary processes, while in Section 5, we consider the case of processes that may be integrated. In Section 6, we illustrate the procedures on a monthly VAR model of the U.S. economy involving a monetary variable (nonborrowed reserves), an interest rate (federal funds rate), prices (GDP deflator) and real GDP, over the period 1965–1996. We conclude in Section 7.

2. Multiple horizon autoregressions

In this section, we develop the notion of "autoregression at horizon h" and the relevant notations. Consider a VAR(p) process of the form:

$$W(t) = \mu(t) + \sum_{k=1}^{p} \pi_k W(t-k) + a(t), \quad t = 1, \ldots, T, \tag{1}$$

where $W(t) = (w_{1t}, w_{2t}, \ldots, w_{mt})'$ is an $m \times 1$ random vector, $\mu(t)$ is a deterministic trend, and

$$\begin{aligned} \mathsf{E}[a(s)\,a(t)'] &= \Omega \quad \text{if } s = t, \\ &= 0 \quad \text{if } s \neq t, \qquad (2) \\ \det(\Omega) &\neq 0. \qquad (3) \end{aligned}$$

The most common specification for $\mu(t)$ consists in assuming that $\mu(t)$ is a constant vector, i.e.

$$\mu(t) = \mu, \tag{4}$$

although other deterministic trends could also be considered.

The VAR(p) in Eq. (1) is an autoregression at horizon 1. We can then also write for the observation at time $t+h$:

$$W(t+h) = \mu^{(h)}(t) + \sum_{k=1}^{p} \pi_k^{(h)} W(t+1-k) + \sum_{j=0}^{h-1} \psi_j a(t+h-j),$$
$$t = 0, \ldots, T-h,$$

where $\psi_0 = I_m$ and $h < T$. The appropriate formulas for the coefficients $\pi_k^{(h)}$, $\mu^{(h)}(t)$ and ψ_j are given in Dufour and Renault (1998), namely

$$\pi_k^{(h+1)} = \pi_{k+h} + \sum_{l=1}^{h} \pi_{h-l+1}\pi_k^{(l)} = \pi_{k+1}^{(h)} + \pi_1^{(h)}\pi_k, \quad \pi_1^{(0)} = I_m, \quad \pi_k^{(1)} = \pi_k, \tag{5}$$

$$\mu^{(h)}(t) = \sum_{k=0}^{h-1} \pi_1^{(k)} \mu(t+h-k), \quad \psi_h = \pi_1^{(h)}, \quad \forall h \geqslant 0. \tag{6}$$

The ψ_h matrices are the impulse response coefficients of the process, which can also be obtained from the formal series:

$$\psi(z) = \pi(z)^{-1} = I_m + \sum_{k=1}^{\infty} \psi_k z^k, \quad \pi(z) = I_m - \sum_{k=1}^{\infty} \pi_k z^k. \tag{7}$$

Equivalently, the above equation for $W(t+h)$ can be written in the following way:

$$\begin{aligned} W(t+h)' &= \mu^{(h)}(t)' + \sum_{k=1}^{p} W(t+1-k)'\pi_k^{(h)\prime} + u^{(h)}(t+h)' \\ &= \mu^{(h)}(t)' + W(t,p)'\pi^{(h)} + u^{(h)}(t+h)', \quad t = 0, \ldots, T-h, \qquad (8) \end{aligned}$$

where $W(t,p)' = [W(t)', W(t-1)', \ldots, W(t-p+1)']$, $\pi^{(h)} = [\pi_1^{(h)}, \ldots, \pi_p^{(h)}]'$ and

$$u^{(h)}(t+h)' = [u_1^{(h)}(t+h), \ldots, u_m^{(h)}(t+h)] = \sum_{j=0}^{h-1} a(t+h-j)'\psi_j'.$$

It is straightforward to see that $u^{(h)}(t+h)$ has a non-singular covariance matrix.

We call (8) an "autoregression of order p at horizon h" or a "(p, h)-autoregression". In the sequel, we will assume that the deterministic part of each autoregression is a linear function of a finite-dimensional parameter vector, i.e.

$$\mu^{(h)}(t) = \gamma(h) D^{(h)}(t), \tag{9}$$

where $\gamma(h)$ is a $m \times n$ coefficient vector and $D^{(h)}(t)$ is a $n \times 1$ vector of deterministic regressors. If $\mu(t)$ is a constant vector, i.e. $\mu(t) = \mu$, then $\mu^{(h)}(t)$ is simply a constant vector (which may depend on h):

$$\mu^{(h)}(t) = \mu_h. \tag{10}$$

To derive inference procedures, it will be useful to put (8) in matrix form, which yields

$$w_h(h) = \overline{W}_p(h) \Pi^{(h)} + U_h(h), \quad h = 1, \ldots, H, \tag{11}$$

where $w_h(k)$ and $U_h(k)$ are $(T - k + 1) \times m$ matrices and $\overline{W}_p(k)$ is a $(T - k + 1) \times (n + mp)$ matrix defined as

$$w_h(k) = \begin{bmatrix} W(0+h)' \\ W(1+h)' \\ \vdots \\ W(T-k+h)' \end{bmatrix} = [w_1(h, k), \ldots, w_m(h, k)], \tag{12}$$

$$\overline{W}_p(k) = \begin{bmatrix} W_p(0)' \\ W_p(1)' \\ \vdots \\ W_p(T-k)' \end{bmatrix}, \quad W_p(t) = \begin{bmatrix} D^{(h)}(t)' \\ W(t, p) \end{bmatrix}, \tag{13}$$

$$\Pi^{(h)} = \begin{bmatrix} \gamma(h)' \\ \pi^{(h)} \end{bmatrix} = \left[\beta_1(h), \beta_2(h), \ldots, \beta_m(h)\right], \tag{14}$$

$$U_h(k) = \begin{bmatrix} u^{(h)}(0+h)' \\ u^{(h)}(1+h)' \\ \vdots \\ u^{(h)}(T-k+h)' \end{bmatrix} = [u_1(h, k), \ldots, u_m(h, k)], \tag{15}$$

$$u_i(h, k) = [u_i^{(h)}(0+h), u_i^{(h)}(1+h), \ldots, u_i^{(h)}(T-k+h)]'. \tag{16}$$

We shall call the formulation (11) a "(p, h)-autoregression in matrix form".

We shall call the formulation (11) a "(p, h)-autoregression in matrix form". Other formulations could also be written by stacking autoregressions at different horizons; see the discussion paper version of this article (Dufour et al., 2003).

3. Estimation of (p, h) autoregressions

Let us now consider each autoregression of order p at horizon h as given by (11)

$$w_h(h) = \overline{W}_p(h)\Pi^{(h)} + U_h(h), \quad h = 1, \dots, H. \tag{17}$$

We can estimate (17) by ordinary least squares (OLS), which yields the estimator

$$\hat{\Pi}^{(h)} = [\overline{W}_p(h)'\overline{W}_p(h)]^{-1}\overline{W}_p(h)'w_h(h) = \Pi^{(h)} + [\overline{W}_p(h)'\overline{W}_p(h)]^{-1}\overline{W}_p(h)'U_h(h),$$

hence

$$\sqrt{T}[\hat{\Pi}^{(h)} - \Pi^{(h)}] = \left[\frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h)\right]^{-1} \frac{1}{\sqrt{T}}\,\overline{W}_p(h)'U_h(h),$$

where

$$\frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h) = \frac{1}{T}\sum_{t=0}^{T-h} W_p(t)W_p(t)',$$

$$\frac{1}{\sqrt{T}}\,\overline{W}_p(h)'U_h(h) = \frac{1}{\sqrt{T}}\sum_{t=0}^{T-h} W_p(t)u^{(h)}(t+h)'.$$

Suppose now that

$$\frac{1}{T}\sum_{t=0}^{T-h} W_p(t)W_p(t)' \xrightarrow[T\to\infty]{\mathsf{p}} \Gamma_p \quad \text{with } \det(\Gamma_p) \neq 0. \tag{18}$$

In particular, this will be the case if the process $W(t)$ is second-order stationary, strictly indeterministic and regular, in which case

$$\mathsf{E}[W_p(t)W_p(t)'] = \Gamma_p, \quad \forall t. \tag{19}$$

Cases where the process does not satisfy these conditions are covered in Section 5. Further, since

$$u^{(h)}(t+h) = a(t+h) + \sum_{k=1}^{h-1} \psi_k a(t+h-k),$$

(where, by convention, any sum of the form $\sum_{k=1}^{h-1}$ with $h < 2$ is zero), we have

$$\mathsf{E}[W_p(t)u^{(h)}(t+h)'] = 0 \quad \text{for } h = 1, 2, \dots,$$
$$\mathsf{V}\{vec[W_p(t)u^{(h)}(t+h)']\} = \Delta_p(h).$$

If the process $W(t)$ is strictly stationary with i.i.d. innovations $a(t)$ and finite fourth moments, we can write

$$\mathsf{E}[W_p(s)u_i^{(h)}(s+h)u_j^{(h)}(t+h)W_p(t)'] = \Gamma_{ij}(p, h, t-s) = \Gamma_{ij}(p, h, s-t), \tag{20}$$

where $1 \leqslant i \leqslant m$, $1 \leqslant j \leqslant m$, with

$$\begin{aligned} \Gamma_{ij}(p,h,0) &= \mathsf{E}[W_p(t)u_i^{(h)}(t+h)u_j^{(h)}(t+h)W_p(t)'] \\ &= \sigma_{ij}(h)\mathsf{E}[W_p(t)W_p(t)'] = \sigma_{ij}(h)\Gamma_p, \end{aligned} \tag{21}$$

$$\Gamma_{ij}(p,h,t-s) = 0 \quad \text{if } |t-s| \geqslant h. \tag{22}$$

In this case,[1]

$$\Delta_p(h) = [\sigma_{ij}(h)\Gamma_p]_{i,j=1,\dots,m} = \Sigma(h) \otimes \Gamma_p, \tag{23}$$

where $\Sigma(h)$ is nonsingular, and thus $\Delta_p(h)$ is also nonsingular. The nonsingularity of $\Sigma(h)$ follows from the identity

$$u^{(h)}(t+h) = [\psi_{h-1}, \psi_{h-2}, \dots, \psi_1, I_m][a(t+1)', a(t+2)', \dots, a(t+h)']'.$$

Under usual regularity conditions,

$$\frac{1}{\sqrt{T}} \sum_{t=0}^{T-h} vec[W_p(t)u^{(h)}(t+h)'] \xrightarrow[T\to\infty]{\mathsf{L}} N[0, \bar{\Delta}_p(h)], \tag{24}$$

where $\bar{\Delta}_p(h)$ is a nonsingular covariance matrix which involves the variance and the autocovariances of $W_p(t)u^{(h)}(t+h)'$ [and possibly other parameters, if the process $W(t)$ is not linear]. Then,

$$\begin{aligned} \sqrt{T}vec[\hat{\Pi}^{(h)} - \Pi^{(h)}] &= \left\{ I_m \otimes \left[\frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h)\right]^{-1} \right\} vec\left[\frac{1}{\sqrt{T}}\,\overline{W}_p(h)'U_h(h)\right] \\ &= \left\{ I_m \otimes \left[\frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h)\right]^{-1} \right\} \frac{1}{\sqrt{T}} \\ &\quad \times \sum_{t=0}^{T-h} vec[W_p(t)u^{(h)}(t+h)'] \\ &\quad \xrightarrow[T\to\infty]{\mathsf{L}} N[0, (I_m \otimes \Gamma_p^{-1})\bar{\Delta}_p(h)(I_m \otimes \Gamma_p^{-1})]. \end{aligned} \tag{25}$$

For convenience, we shall summarize the above observations in the following proposition.

Proposition 1. Asymptotic normality of LS in a (p,h) stationary VAR. *Under the assumptions* (1), (18), *and* (24), *the asymptotic distribution of* $\sqrt{T}\,vec[\hat{\Pi}^{(h)} - \Pi^{(h)}]$ *is* $N[0, \Sigma(\hat{\Pi}^{(h)})]$, *where* $\Sigma(\hat{\Pi}^{(h)}) = (I_m \otimes \Gamma_p^{-1})\bar{\Delta}_p(h)(I_m \otimes \Gamma_p^{-1})$.

[1]Note that (21) holds under the assumption of martingale difference sequence on $a(t)$. But to get (22) and allow the use of simpler central limit theorems, we maintain the stronger assumption that the innovations $a(t)$ are i.i.d. according to some distribution with finite fourth moments (not necessarily Gaussian).

4. Causality tests based on stationary (p, h)-autoregressions

Consider the ith equation $(1 \leqslant i \leqslant m)$ in system (11):

$$\bar{w}_i(h) = \overline{W}_p(h)\beta_i(h) + \bar{u}_i(h), \quad 1 \leqslant i \leqslant m, \tag{26}$$

where $\bar{w}_i(h) = w_i(h, h)$ and $\bar{u}_i(h) = u_i(h, h)$, where $w_i(h, h)$ and $u_i(h, h)$ are defined in (12) and (15). We wish to test

$$\mathrm{H}_0(h) : R\beta_i(h) = r, \tag{27}$$

where R is a $q \times (n + mp)$ matrix of rank q. In particular, if we wish to test the hypothesis that w_{jt} does not cause w_{it} at horizon h [i.e., using the notation of Dufour and Renault (1998), $w_j \underset{h}{\nrightarrow} w_i \,|\, I_{(j)}$, where $I_{(j)}(t)$ is the Hilbert space generated by the basic information set $I(t)$ and the variables $w_{k\tau}, \omega < \tau \leqslant t,\ k \neq j,\ \omega$ being an appropriate starting time $(\omega \leqslant -p + 1)$], the restriction would take the form:

$$H^{(h)}_{j \nrightarrow i} : \pi^{(h)}_{ijk} = 0, \quad k = 1, \ldots, p, \tag{28}$$

where $\pi^{(h)}_k = [\pi^{(h)}_{ijk}]_{i,\ j=1,\ldots,m}$, $k = 1, \ldots, p$. In other words, the null hypothesis takes the form of a set of zero restrictions on the coefficients of $\beta_i(h)$ as defined in (14). The matrix of restrictions R in this case takes the form $R = R(j)$, where $R(j) \equiv [\delta_1(j), \delta_2(j), \ldots, \delta_p(j)]'$ is a $p \times (n + mp)$ matrix, $\delta_k(j)$ is a $(n + pm) \times 1$ vector whose elements are all equal to zero except for a unit value at position $n + (k - 1)m + j$, i.e. $\delta_k(j) = [\delta(1, n + (k - 1)m + j), \ldots, \delta(n + pm, n + (k - 1)m + j)]'$, $k = 1, \ldots, p$, with $\delta(i,j) = 1$ if $i = j$, and $\delta(i, j) = 0$ if $i \neq j$. Note also that the conjunction of the hypothesis $H^{(h)}_{j \nrightarrow i}$, $h = 1, \ldots, (m - 2)p + 1$, is sufficient to obtain noncausality at all horizons [see (Dufour and Renault, 1998, Section 4)]. Non-causality up to horizon H is the conjunction of the hypothesis $H^{(h)}_{j \nrightarrow i}$, $h = 1, \ldots, H$.

We have

$$\hat{\beta}_i(h) = \beta_i(h) + [\,\overline{W}_p(h)'\overline{W}_p(h)]^{-1}\overline{W}_p(h)'\bar{u}_i(h),$$

hence

$$\sqrt{T}\,[\hat{\beta}_i(h) - \beta_i(h)] = \left[\frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h)\right]^{-1} \frac{1}{\sqrt{T}} \sum_{t=0}^{T-h} W_p(t)u^{(h)}_i(t + h).$$

Under standard regularity conditions [see White, 1999, Chapter 5–6],

$$\sqrt{T}\,[\hat{\beta}_i(h) - \beta_i(h)] \xrightarrow[T\to\infty]{L} N[0,\, \mathsf{V}(\hat{\beta}_i)]$$

with $\det[\mathsf{V}(\hat{\beta}_i)] \neq 0$, where $\mathsf{V}(\hat{\beta}_i)$ can be consistently estimated:

$$\hat{V}_T(\hat{\beta}_i) \xrightarrow[T\to\infty]{\mathsf{p}} \mathsf{V}(\hat{\beta}_i).$$

More explicit forms for $\hat{V}_T(\hat{\beta}_i)$ will be discussed below. Note also that

$$\Gamma_p = \operatorname*{plim}_{T\to\infty} \frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h), \quad \det(\Gamma_p) \neq 0.$$

Let

$$V_{ip}(T) = \mathsf{Var}\left[\frac{1}{\sqrt{T}}\,\overline{W}_p(h)'\bar{u}_i(h)\right] = \frac{1}{T}\,\mathsf{Var}\left[\sum_{t=0}^{T-h} W_p(t)u_i^{(h)}(t+h)\right]$$
$$= \frac{1}{T}\left\{\sum_{t=0}^{T-h} \mathsf{E}[W_p(t)u_i^{(h)}(t+h)u_i^{(h)}(t+h)W_p(t)'] + \sum_{\tau=1}^{h-1}\sum_{t=\tau+1}^{T-h}[\mathsf{E}[W_p(t)u_i^{(h)}(t+h)u_i^{(h)}(t-\tau+h)W_p(t-\tau)'] + \mathsf{E}[W_p(t-\tau)u_i^{(h)}(t-\tau+h)u_i^{(h)}(t+h)W_p(t)']]\right\}.$$

Let us assume that

$$V_{ip}(T) \underset{T\to\infty}{\longrightarrow} V_{ip}, \quad \det(V_{ip}) \neq 0, \tag{29}$$

where V_{ip} can be estimated by a computable consistent estimator $\hat{V}_{ip}(T)$:

$$\hat{V}_{ip}(T) \underset{T\to\infty}{\overset{\mathsf{p}}{\longrightarrow}} V_{ip}. \tag{30}$$

Then,

$$\sqrt{T}[\hat{\beta}_i(h) - \beta_i(h)] \underset{T\to\infty}{\overset{\mathsf{p}}{\longrightarrow}} N[0,\, \Gamma_p^{-1}V_{ip}\Gamma_p^{-1}],$$

so that $\mathsf{V}(\hat{\beta}_i) = \Gamma_p^{-1}V_{ip}\Gamma_p^{-1}$. Further, in this case,

$$\hat{V}_T(\hat{\beta}_i) = \hat{\Gamma}_p^{-1}\hat{V}_{ip}(T)\hat{\Gamma}_p^{-1} \underset{T\to\infty}{\overset{\mathsf{p}}{\longrightarrow}} \mathsf{V}(\hat{\beta}_i),$$
$$\hat{\Gamma}_p = \frac{1}{T}\sum_{t=0}^{T-h} W_p(t)W_p(t)' = \frac{1}{T}\,\overline{W}_p(h)'\overline{W}_p(h) \underset{T\to\infty}{\overset{\mathsf{p}}{\longrightarrow}} \Gamma_p.$$

We can thus state the following proposition.

Proposition 2. Asymptotic distribution of test criterion for non-causality at horizon h in a stationary VAR. *Suppose the assumptions of Proposition* 1 *hold jointly with* (29)–(30). *Then, under any hypothesis of the form* $\mathrm{H}_0(h)$ *in* (27), *the asymptotic distribution of*

$$\mathscr{W}[\mathrm{H}_0(h)] = T\,[R\hat{\beta}_i(h) - r]'[R\hat{V}_T(\hat{\beta}_i)R']^{-1}[R\hat{\beta}_i(h) - r] \tag{31}$$

is $\chi^2(q)$. *In particular, under the hypothesis* $H_{j \nrightarrow i}^{(h)}$ *of non-causality at horizon h from* w_{jt} *to* w_{it} $(w_j \underset{h}{\nrightarrow} w_i \mid I_{(j)})$, *the asymptotic distribution of the corresponding statistic* $\mathscr{W}[\mathrm{H}_0(h)]$ *is* $\chi^2(p)$.

The problem now consists in estimating V_{ip}. Let $\hat{\bar{u}}_i(h) = [\hat{u}_i^{(h)}(t+h) : t = 0, \ldots, T-h]'$ be the vector of OLS residuals from the regression (26),

$\hat{g}_i^{(h)}(t+h) = W_p(t)\hat{u}_i^{(h)}(t+h)$, and set

$$R_i^{(h)}(\tau) = \frac{1}{T-h}\sum_{t=\tau}^{T-h} \hat{g}_i^{(h)}(t+h)\hat{g}_i^{(h)}(t+h-\tau)', \quad \tau = 0, 1, 2, \ldots .$$

If the innovations are i.i.d. or, more generally, if (22) holds, a natural estimator of V_{ip}, which would take into account the fact that the prediction errors $u^{(h)}(t+h)$ follow an MA$(h-1)$ process, is given by

$$\hat{V}_{ip}^{(W)}(T) = R_i^{(h)}(0) + \sum_{\tau=1}^{h-1}[R_i^{(h)}(\tau) + R_i^{(h)}(\tau)'].$$

Under regularity conditions studied by White (1999, Section 6.3),

$$\hat{V}_{ip}^{(W)}(T) - V_{ip} \underset{T\to\infty}{\overset{\mathrm{p}}{\longrightarrow}} 0.$$

A problem with $\hat{V}_{ip}^{(W)}(T)$ is that it is not necessarily positive-definite.

An alternative estimator which is automatically positive-semidefinite is the one suggested by Doan and Litterman (1983), Gallant (1987) and Newey and West (1987):

$$\hat{V}_{ip}^{(\mathrm{NW})}(T) = R_i^{(h)}(0) + \sum_{\tau=1}^{m(T)-1} \kappa(\tau, m(T))\,[R_i^{(h)}(\tau) + R_i^{(h)}(\tau)'], \tag{32}$$

where $\kappa(\tau, m) = 1 - [\tau/(m+1)]$, $\lim_{T\to\infty} m(T) = \infty$, and $\lim_{T\to\infty}[m(T)/T^{1/4}] = 0$. Under the regularity conditions given by Newey and West (1987),

$$\hat{V}_{ip}^{(\mathrm{NW})}(T) - V_{ip} \underset{T\longrightarrow\infty}{\longrightarrow} 0.$$

Other estimators that could be used here includes various HAC estimators; see Andrews (1991), Andrews and Monahan (1992), Cribari-Neto et al. (2000), Cushing and McGarvey (1999), Den Haan and Levin (1997), Hansen (1992), Newey and McFadden (1994), Wooldridge (1989).

The cost of having a simple procedure that sidestep all the nonlinearities associated with the non-causality hypothesis is a loss of efficiency. There are two places where we are not using all information. The constraints on the $\pi_k^{(h)}$'s are giving information on the ψ_j's and we are not using it. We are also estimating the VAR by OLS and correcting the variance–covariance matrix instead of doing a GLS-type estimation. These two sources of inefficiencies could potentially be overcome but it would lead to less user-friendly procedures.

The asymptotic distribution provided by Proposition 2, may not be very reliable in finite samples, especially if we consider a VAR system with a large number of variables and/or lags. Due to autocorrelation, a larger horizon may also affect the size and power of the test. So an alternative to using the asymptotic distribution chi-square of $\mathscr{W}[\mathrm{H}_0(h)]$, consists in using Monte Carlo test techniques [see (Dufour, 2002)] or bootstrap methods [see, for example, Paparoditis (1996), Paparoditis and Streitberg (1991), Kilian (1998a, b)]. In view of the fact that the asymptotic distribution of $\mathscr{W}[\mathrm{H}_0(h)]$ is nuisance-parameter-free, such methods yield asymptotically valid tests when applied to $\mathscr{W}[\mathrm{H}_0(h)]$ and typically provide a much better control of test level in finite samples. It is also possible that using better estimates would improve size control, although this is not clear, for important size distortions can occur in multivariate regressions even when unbiased efficient estimators are available [see, for example, Dufour and Khalaf (2002)].

5. Causality tests based on nonstationary (p, h)-autoregressions

In this section, we study how the tests described in the previous section can be adjusted in order to allow for non-stationary possibly integrated processes. In particular, let us assume that

$$W(t) = \mu(t) + \eta(t), \tag{33}$$

$$\mu(t) = \delta_0 + \delta_1 t + \cdots + \delta_q t^q, \quad \eta(t) = \sum_{k=1}^{p} \pi_k \eta(t-k) + a(t), \tag{34}$$

$t = 1, \ldots, T$, where $\delta_0,\ \delta_1, \ldots, \delta_q$ are $m \times 1$ fixed vectors, and the process $\eta(t)$ is at most $I(d)$ where d is an integer greater than or equal to zero. Typical values for d are 0, 1 or 2. Note that these assumptions allow for the presence (or the absence) of cointegration relationships.

Under the above assumptions, we can also write

$$W(t) = \gamma_0 + \gamma_1 t + \cdots + \gamma_q t^q + \sum_{k=1}^{p} \pi_k W(t-k) + a(t), \quad t = 1, \ldots, T, \tag{35}$$

where $\gamma_0,\ \gamma_1, \ldots, \gamma_q$ are $m \times 1$ fixed vectors (which depend on $\delta_0,\ \delta_1, \ldots, \delta_q$, and $\pi_1, \ldots, \pi_p$); see Toda and Yamamoto (1995). Under the specification (35), we have

$$W(t+h) = \mu^{(h)}(t) + \sum_{k=1}^{p} \pi_k^{(h)} W(t+1-k) + u^{(h)}(t+h), \quad t = 0, \ldots, T-h, \tag{36}$$

where $\mu^{(h)}(t) = \gamma_0^{(h)} + \gamma_1^{(h)} t + \cdots + \gamma_q^{(h)} t^q$ and $\gamma_0^{(h)},\ \gamma_1^{(h)}, \ldots, \gamma_q^{(h)}$ are $m \times 1$ fixed vectors. For $h = 1$, this equation is identical with (35). For $h \geqslant 2$, the errors $u^{(h)}(t+h)$ follow a MA$(h-1)$ process as opposed to being i.i.d.. For any integer j, we have:

$$\begin{aligned} W(t+h) = {} & \mu^{(h)}(t) + \sum_{\substack{k=1 \\ k \neq j}}^{p} \pi_k^{(h)} [W(t+1-k) - W(t+1-j)] \\ & + \left(\sum_{k=1}^{p} \pi_k^{(h)} \right) W(t+1-j) + u^{(h)}(t+h), \end{aligned} \tag{37}$$

$$W(t+h) - W(t+1-j) = \mu^{(h)}(t) + \sum_{\substack{k=1 \\ k \neq j}}^{p} \pi_k^{(h)}[W(t+1-k) - W(t+1-j)] - \left(I_m - \sum_{k=1}^{p} \pi_k^{(h)}\right) W(t+1-j) + u^{(h)}(t+h) \quad (38)$$

for $t = 0, \ldots, T-h$. The two latter expressions can be viewed as extensions to (p, h)-autoregressions of the representations used by Dolado and Lütkepohl (1996, pp. 372–373) for VAR(p) processes. Further, on taking $j = p+1$ in (38), we see that

$$W(t+h) - W(t-p) = \mu^{(h)}(t) + \sum_{k=1}^{p} A_k^{(h)} \Delta W(t+1-k) + B_p^{(h)} W(t-p) + u^{(h)}(t+h), \quad (39)$$

where $\Delta W(t) = W(t) - W(t-1)$, $A_k^{(h)} = \sum_{j=1}^{k} \pi_k^{(h)}$, and $B_k^{(h)} = A_k^{(h)} - I_m$. Eq. (39) may be interpreted as an error-correction form at the horizon h, with base $W(t-p)$.

Let us now consider the extended autoregression

$$W(t+h) = \mu^{(h)}(t) + \sum_{k=1}^{p} \pi_k^{(h)} W(t+1-k) + \sum_{k=p+1}^{p+d} \pi_k^{(h)} W(t+1-k) + u^{(h)}(t+h), \quad (40)$$

$t = d, \ldots, T-h$. Under model (35), the actual values of the coefficient matrices $\pi_{p+1}^{(h)}, \ldots, \pi_{p+d}^{(h)}$ are equal to zero ($\pi_{p+1}^{(h)} = \cdots = \pi_{p+d}^{(h)} = 0$), but we shall estimate the (p, h)-autoregressions without imposing any restriction on $\pi_{p+1}^{(h)}, \ldots, \pi_{p+d}^{(h)}$.

Now, suppose the process $\eta(t)$ is either $I(0)$ or $I(1)$, and we take $d = 1$ in (40). Then, on replacing p by $p+1$ and setting $j = p$ in the representation (38), we see that

$$W(t+h) - W(t-p-1) = \mu^{(h)}(t) + \sum_{k=1}^{p} \pi_k^{(h)}[W(t+1-k) - W(t-p-1)] - B_{p+1}^{(h)} W(t-p-1) + u^{(h)}(t+h), \quad (41)$$

where $B_{p+1}^{(h)} = (I_m - \sum_{k=1}^{p+1} \pi_k^{(h)})$. In the latter equation, $\pi_1^{(h)}, \ldots, \pi_p^{(h)}$ all affect trend-stationary variables (in an equation where a trend is included along with the other coefficients). Using arguments similar to those of Sims et al. (1990), Park and Phillips (1989) and Dolado and Lütkepohl (1996), it follows that the estimates of $\pi_1^{(h)}, \ldots, \pi_p^{(h)}$ based on estimating (41) by ordinary least squares (without restricting $B_{p+1}^{(h)}$) _ or, equivalently, those obtained from (40) without restricting $\pi_{p+1}^{(h)}$ _ are asymptotically normal with the same asymptotic covariance matrix as the one obtained for a stationary process of the type studied in Section 4.[2] Consequently, the asymptotic distribution of the statistic $\mathscr{W}[H_{j \nrightarrow i}^{(h)}]$ for testing the null hypothesis $H_{j \nrightarrow i}^{(h)}$ of

[2] For related results, see also Choi (1993), Toda and Yamamoto (1995), Yamamoto (1996), Yamada and Toda (1998), Kurozumi and Yamamoto (2000).

non-causality at horizon h from w_j to $w_i(w_j \underset{h}{\nrightarrow} w_i \mid I_{(j)})$, based on estimating (40), is $\chi^2(p)$. When computing $H^{(h)}_{j\nrightarrow i}$ as defined in (28), it is important that only the coefficients of $\pi^{(h)}_1, \ldots, \pi^{(h)}_p$ are restricted (but not $\pi^{(h)}_{p+1}$).

If the process $\eta(t)$ is integrated up to order d, where $d \geqslant 0$, we can proceed similarly and add d extra lags to the VAR process studied. Again, the null hypothesis is tested by considering the restrictions entailed on $\pi^{(h)}_1, \ldots, \pi^{(h)}_p$. Further, in view of the fact the test statistics are asymptotically pivotal under the null hypothesis, it is straightforward to apply bootstrap methods to such statistics. Note finally that the precision of the VAR estimates in such augmented regressions may eventually be improved with respect to the OLS estimates considered here by applying bias corrections such as those proposed by Kurozumi and Yamamoto (2000). Adapting and applying such corrections to (p, h)-autoregressions would go beyond the scope of the present paper.

6. Empirical illustration

In this section, we present an application of these causality tests at various horizons to macroeconomic time series. The data set considered is the one used by Bernanke and Mihov (1998) in order to study United States monetary policy. The data set considered consists of monthly observations on nonborrowed reserves (NBR, also denoted w_1), the federal funds rate (r, w_2), the GDP deflator (P, w_3) and real GDP (GDP, w_4). The monthly data on GDP and GDP deflator were constructed by state space methods from quarterly observations [see (Bernanke and Mihov, 1998) for more details]. The sample goes from January 1965 to December 1996 for a total of 384 observations. In what follows, all the variables were first transformed by a logarithmic transformation.

Before performing the causality tests, we must specify the order of the VAR model. First, in order to get apparently stationary time series, all variables were transformed by taking first differences of their logarithms. In particular, for the federal funds rate, this helped to mitigate the effects of a possible break in the series in the years 1979–1981.[3] Starting with 30 lags, we then tested the hypothesis of K lags versus $K+1$ lags using the LR test presented in Tiao and Box (1981). This led to a VAR(16) model. Tests of a VAR(16) against a VAR(K) for $K = 17, \ldots, 30$ also failed to reject the VAR(16) specification, and the AIC information criterion [see McQuarrie and Tsai, 1998, Chapter 5)] is minimized as well by this choice. Calculations were performed using the Ox program (version 3.00) working on Linux [see (Doornik, 1999)].

Vector autoregressions of order p at horizon h were estimated as described in Section 4 and the matrix $\hat{V}^{(\mathrm{NW})}_{ip}$, required to obtain covariance matrices, were

[3] Bernanke and Mihov (1998) performed tests for arbitrary break points, as in Andrews (1993), and did not find significant evidence of a break point. They considered a VAR(13) with two additional variables (total bank reserves and Dow–Jones index of spot commodity prices and they normalize both reserves by a 36-month moving average of total reserves.)

Table 1
Rejection frequencies using the asymptotic distribution and the bootstrap procedure

$h=$	1	2	3	4	5	6	7	8	9	10	11	12
(a) *i.i.d. Gaussian sequence*												
Asymptotic												
5% level	27.0	27.8	32.4	36.1	35.7	42.6	47.9	48.5	51.0	55.7	59.7	63.6
10% level	37.4	39.4	42.2	46.5	47.8	52.0	58.1	59.3	60.3	66.3	69.2	72.5
Bootstrap												
5% level	5.5	5.7	4.7	6.5	4.0	5.1	5.5	3.9	4.7	6.1	5.2	3.8
10% level	10.0	9.1	10.1	10.9	9.6	10.6	10.2	9.4	9.5	10.9	10.3	8.9
(b) *VAR(16) without causality up to horizon h*												
Asymptotic												
5% level	24.1	27.9	35.8	37.5	55.9	44.3	52.3	55.9	54.1	60.1	62.6	72.0
10% level	35.5	38.3	46.6	47.2	65.1	55.0	64.7	64.6	64.8	69.8	72.0	79.0
Bootstrap												
5% level	6.0	5.1	3.8	6.1	4.6	4.7	4.4	4.5	4.3	6.3	4.9	5.8
10% level	9.8	8.8	8.7	10.4	10.3	9.9	8.7	7.4	10.3	11.1	9.3	9.7

computed using formula (32) with $m(T) - 1 = h - 1$.[4] On looking at the values of the test statistics and their corresponding p-values at various horizons it quickly becomes evident that the $\chi^2(q)$ asymptotic approximation of the statistic $\mathscr{W}$ in Eq. (31) is very poor. As a simple Monte Carlo experiment, we replaced the data by a 383×4 matrix of random draw from an $N(0,1)$, ran the same tests and looked at the rejection frequencies over 1000 replications using the asymptotic critical value. The results are in Table 1a. We see important size distortions even for the tests at horizon 1 where there is no moving average part.

We next illustrate that the quality of the asymptotic approximation is even worse when we move away from an i.i.d. Gaussian setup to a more realistic case. We now take as the DGP the VAR(16) estimated with our data in first difference but we impose that some coefficients are zero such that the federal funds rate does not cause GDP up to horizon h and then we test the $r \underset{h}{\nrightarrow} GDP$ hypothesis. The constraints of

[4]The covariance estimator used here is relatively simple and exploits the truncation property (21). In view of the vast literature on HAC estimators [see Den Haan and Levin (1997), Cushing and McGarvey (1999)], several alternative estimators for V_{ip} could have been considered (possibly allowing for alternative assumptions on the innovation distribution). It would certainly be of interest to compare the performances of alternative covariance estimators, but this would lead to a lengthy study, beyond the scope of the present paper.

non-causality from j to i up to horizon h that we impose are

$$\hat{\pi}_{ijl} = 0 \quad \text{for } 1 \leqslant l \leqslant p, \tag{42}$$

$$\hat{\pi}_{ikl} = 0 \quad \text{for } 1 \leqslant l \leqslant h, \ 1 \leqslant k \leqslant m. \tag{43}$$

Rejection frequencies for this case are given in Table 1b.

In light of these results we computed the p-values by doing a parametric bootstrap, i.e. doing an asymptotic Monte Carlo test based on a consistent point estimate [see (Dufour, 2002)]. The procedure to test the hypothesis $w_j \underset{h}{\nrightarrow} w_i \mid I_{(j)}$ is the following:

1. An unrestricted VAR(p) model is fitted for the horizon one, yielding the estimates $\hat{\Pi}^{(1)}$ and $\hat{\Omega}$ for $\Pi^{(1)}$ and Ω.
2. An unrestricted (p, h)-autoregression is fitted by least squares, yielding the estimate $\hat{\Pi}^{(h)}$ of $\Pi^{(h)}$.
3. The test statistic $\mathscr{W}$ for testing noncausality at the horizon h from w_j to w_i $[H^{(h)}_{j \nrightarrow i} : w_j \underset{h}{\nrightarrow} w_i \mid I_{(j)}]$ is computed. We denote by $\mathscr{W}^{(h)}_{j \nrightarrow i}(0)$ the test statistic based on the actual data.
4. N simulated samples from (8) are drawn by Monte Carlo methods, using $\Pi^{(h)} = \hat{\Pi}^{(h)}$ and $\Omega = \hat{\Omega}$ [and the hypothesis that $a(t)$ is Gaussian].We impose the constraints of non-causality, $\hat{\pi}^{(h)}_{ijk} = 0, \ k = 1, \ldots, p$. Estimates of the impulse response coefficients are obtained from $\hat{\Pi}^{(1)}$ through the relations described in Eq. (5). We denote by $\mathscr{W}^{(h)}_{j \nrightarrow i}(n)$ the test statistic for $H^{(h)}_{j \nrightarrow i}$ based on the nth simulated sample $(1 \leqslant n \leqslant N)$.
5. The simulated p-value $\hat{p}_N[\mathscr{W}^{(h)}_{j \nrightarrow i}(0)]$ is obtained, where
$$\hat{p}_N[x] = \left\{ 1 + \sum_{n=1}^{N} I[\mathscr{W}^{(h)}_{j \nrightarrow i}(n) - x] \right\} \Big/ (N+1),$$
$I[z] = 1$ if $z \geqslant 0$ and $I[z] = 0$ if $z < 0$.
6. The null hypothesis $H^{(h)}_{j \nrightarrow i}$ is rejected at level α if $\hat{p}_N[\mathscr{W}^{(0)}_{j \nrightarrow i}(h)] \leqslant \alpha$.

From looking at the results in Table 1, we see that we get a much better size control by using this bootstrap procedure. The rejection frequencies over 1000 replications (with $N = 999$) are very close to the nominal size. Although the coefficients ψ_j's are functions of the π_i's we do not constrain them in the bootstrap procedure because there is no direct mapping from $\pi^{(h)}_k$ to π_k and ψ_j. This certainly produces a power loss but the procedure remains valid because the ψ_j's are computed with the $\hat{\pi}_k$, which are consistent estimates of the true π_k both under the null and alternative hypothesis. To illustrate that our procedure has power for detecting departure from the null hypothesis of non-causality at a given horizon we ran the following Monte Carlo experiment. We again took a VAR(16) fitted on our data in first differences and we imposed the

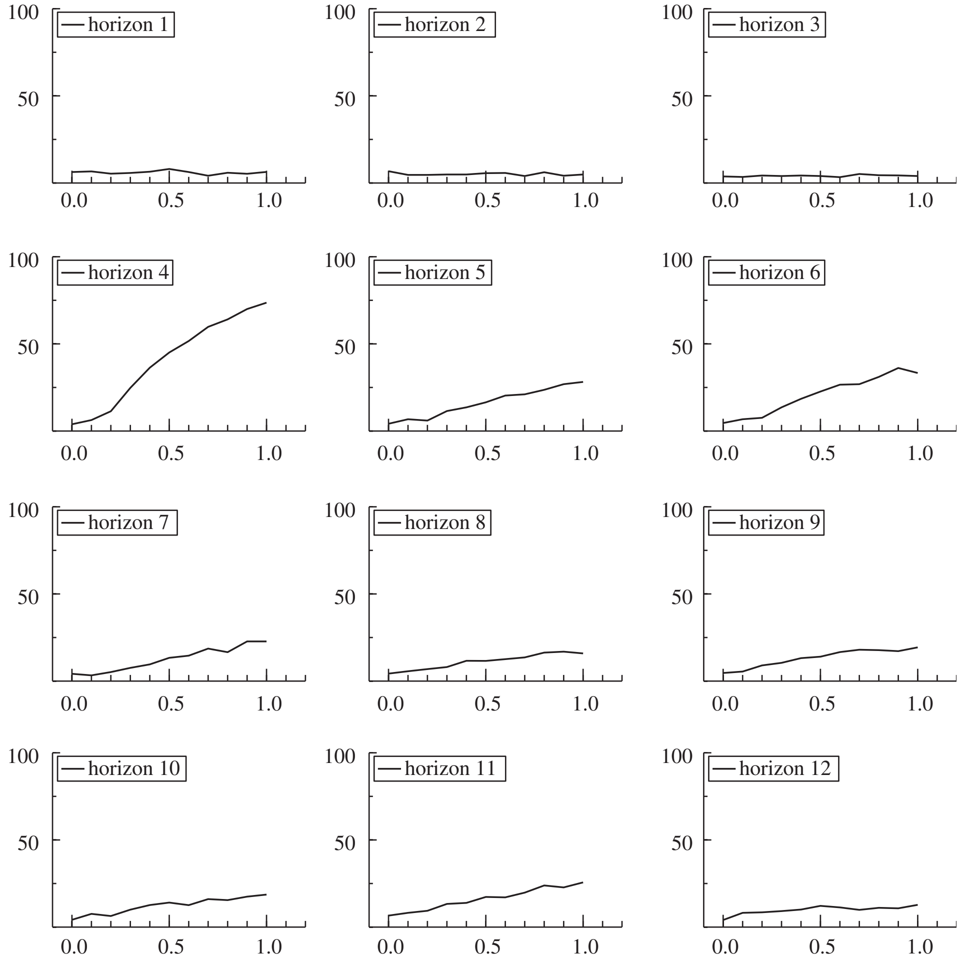

Fig. 1. Power of the test at the 5% level for given horizons. The abscissa (x-axis) represents the values of θ.

constraints (42)–(43) so that there was no causality from r to GDP up to horizon 12 (DGP under the null hypothesis). Next the value of one coefficient previously set to zero was changed to induce causality from r to GDP at horizons 4 and higher: $\pi_3(1, 3) = \theta$. As θ increases from zero to one the strength of the causality from r to GDP is higher. Under this setup, we could compute the power of our simulated test procedure to reject the null hypothesis of non-causality at a given horizon. In Fig. 1, the power curves are plotted as a function of θ for the various horizons. The level of the tests was controlled through the bootstrap procedure. In this experiment we took again $N = 999$ and we did 1000 simulations. As expected, the power curves are flat at around 5% for horizons one to three since the null is true for these horizons. For horizons four and up we get the expectedresult that power goes up as θ moves from zero to one, and the power curves gets flatter as we increase the horizon.

Table 2
Causality tests and simulated p-values for series in first differences (of logarithm) for the horizons 1–12

h	1	2	3	4	5	6	7	8	9	10	11	12
$NBR \nrightarrow r$	38.5205	26.3851	24.3672	22.5684	24.2294	27.0748	21.4347	17.2164	21.8217	18.4775	18.5379	19.6482
	(0.041)	(0.240)	(0.327)	(0.395)	(0.379)	(0.313)	(0.550)	(0.799)	(0.603)	(0.780)	(0.824)	(0.789)
$r \nrightarrow NBR$	22.5390	20.8621	17.2357	17.4222	17.8944	18.6462	25.2059	40.9896	41.6882	38.1656	41.2203	36.1278
	(0.386)	(0.467)	(0.706)	(0.738)	(0.734)	(0.735)	(0.495)	(0.115)	(0.142)	(0.214)	(0.206)	(0.346)
$NBR \nrightarrow P$	50.5547	45.2498	49.1408	33.8545	33.8943	35.3923	39.1767	40.0218	36.4089	39.0916	28.4933	26.5439
	(0.004)	(0.014)	(0.009)	(0.121)	(0.162)	(0.157)	(0.099)	(0.141)	(0.234)	(0.202)	(0.485)	(0.609)
$P \nrightarrow NBR$	17.0696	18.9504	16.6880	21.2381	28.7264	20.0359	18.2291	23.6704	23.3419	27.8427	30.9171	36.5159
	(0.689)	(0.601)	(0.741)	(0.550)	(0.307)	(0.664)	(0.771)	(0.596)	(0.649)	(0.492)	(0.462)	(0.357)
$NBR \nrightarrow GDP$	27.8029	25.0122	25.5123	23.6799	15.0040	17.5748	16.6781	19.6643	29.9020	34.0930	32.3917	34.8183
	(0.184)	(0.302)	(0.294)	(0.393)	(0.830)	(0.757)	(0.790)	(0.746)	(0.368)	(0.300)	(0.370)	(0.357)
$GDP \nrightarrow NBR$	17.6338	20.6568	24.5334	18.3220	18.3123	32.8746	33.9979	39.6701	40.7356	26.3424	40.2262	53.3482
	(0.644)	(0.520)	(0.348)	(0.670)	(0.682)	(0.211)	(0.242)	(0.145)	(0.161)	(0.551)	(0.216)	(0.092)
$r \nrightarrow P$	32.2481	32.9207	32.0362	25.0124	25.2441	25.8110	29.2553	29.6021	36.0771	43.2116	30.2530	20.8982
	(0.104)	(0.108)	(0.138)	(0.342)	(0.383)	(0.394)	(0.328)	(0.351)	(0.241)	(0.138)	(0.422)	(0.763)
$P \nrightarrow r$	22.4385	16.4455	14.3073	14.2932	14.0148	16.7138	11.5599	16.5731	13.0697	14.5759	15.0317	24.4077
	(0.413)	(0.670)	(0.790)	(0.826)	(0.844)	(0.774)	(0.951)	(0.809)	(0.936)	(0.909)	(0.899)	(0.659)
$r \nrightarrow GDP$	26.3362	28.6645	35.9768	38.8680	37.6788	39.8145	64.1500	80.2396	89.6998	101.4143	105.9138	110.3551
	(0.262)	(0.159)	(0.073)	(0.059)	(0.082)	(0.079)	(0.006)	(0.002)	(0.002)	(0.002)	(0.001)	(0.003)
$GDP \nrightarrow r$	42.5327	43.0078	49.8366	41.1082	41.7945	36.0965	29.5472	25.7898	27.9807	39.6366	39.0477	40.1545
	(0.016)	(0.017)	(0.005)	(0.033)	(0.044)	(0.109)	(0.263)	(0.429)	(0.366)	(0.148)	(0.178)	(0.205)
$P \nrightarrow GDP$	20.6903	24.1099	27.4106	23.3585	22.9095	18.5543	20.8172	23.6942	30.5340	28.8286	24.9477	25.7552
	(0.495)	(0.322)	(0.233)	(0.423)	(0.497)	(0.713)	(0.639)	(0.555)	(0.375)	(0.414)	(0.612)	(0.629)
$GDP \nrightarrow P$	24.3368	24.4925	24.6125	22.8160	26.7900	40.0825	36.4855	49.6161	46.2574	36.3197	26.8520	24.0113
	(0.329)	(0.365)	(0.380)	(0.470)	(0.348)	(0.081)	(0.157)	(0.058)	(0.072)	(0.262)	(0.540)	(0.666)

p-values are reported in parenthesis.

Table 3
Causality tests and simulated p-values for series in first differences (of logarithm) for the horizons 13–24

h	13	14	15	16	17	18	19	20	21	22	23	24
$NBR \nrightarrow r$	20.7605	21.1869	18.6062	17.6750	22.2838	34.0098	28.7769	33.7855	66.1538	38.7272	28.8194	34.8015
	(0.783)	(0.771)	(0.852)	(0.905)	(0.811)	(0.532)	(0.696)	(0.632)	(0.143)	(0.558)	(0.803)	(0.708)
$r \nrightarrow NBR$	42.2924	43.8196	32.8113	28.1775	31.2326	31.7587	25.6942	34.6680	37.7470	51.4196	33.2897	27.3362
	(0.245)	(0.254)	(0.533)	(0.707)	(0.625)	(0.651)	(0.832)	(0.682)	(0.625)	(0.406)	(0.755)	(0.869)
$NBR \nrightarrow P$	38.0451	39.0293	43.2101	37.9049	22.4428	20.0337	38.8919	66.1541	55.0460	64.8568	54.8929	42.2509
	(0.336)	(0.351)	(0.315)	(0.413)	(0.835)	(0.883)	(0.488)	(0.149)	(0.255)	(0.180)	(0.332)	(0.562)
$P \nrightarrow NBR$	29.5457	29.4543	27.2425	28.7547	31.4851	44.1254	36.0319	47.1608	50.2743	45.8078	54.1961	56.7865
	(0.565)	(0.667)	(0.688)	(0.703)	(0.627)	(0.400)	(0.581)	(0.427)	(0.379)	(0.486)	(0.401)	(0.399)
$NBR \nrightarrow GDP$	30.3906	25.2694	29.7274	23.3347	16.7784	18.7922	26.8942	30.2550	39.0652	43.6661	56.9724	67.7764
	(0.493)	(0.716)	(0.603)	(0.814)	(0.942)	(0.901)	(0.752)	(0.745)	(0.545)	(0.505)	(0.314)	(0.251)
$GDP \nrightarrow NBR$	44.2264	41.5795	59.3301	65.7647	53.3579	50.4645	51.5163	43.1132	40.7051	36.3960	38.7162	40.8466
	(0.219)	(0.292)	(0.106)	(0.086)	(0.225)	(0.282)	(0.311)	(0.466)	(0.513)	(0.659)	(0.665)	(0.644)
$r \nrightarrow P$	27.3588	37.7170	37.2499	37.1005	27.3052	35.4128	39.2469	53.2675	57.1530	54.6753	69.6377	59.2184
	(0.588)	(0.341)	(0.381)	(0.443)	(0.700)	(0.534)	(0.477)	(0.266)	(0.229)	(0.314)	(0.164)	(0.293)
$P \nrightarrow r$	22.3720	26.3588	28.4930	23.0152	26.4211	36.3632	34.3922	17.4269	16.9652	31.9336	28.8377	30.0013
	(0.750)	(0.608)	(0.633)	(0.794)	(0.747)	(0.512)	(0.536)	(0.956)	(0.958)	(0.715)	(0.801)	(0.795)
$r \nrightarrow GDP$	123.8280	80.3638	95.0727	83.2773	76.6169	84.1068	76.8979	81.3505	65.5025	71.3334	63.1942	64.8020
	(0.001)	(0.018)	(0.005)	(0.018)	(0.033)	(0.040)	(0.055)	(0.073)	(0.161)	(0.153)	(0.241)	(0.286)
$GDP \nrightarrow r$	35.7086	33.0200	40.1713	30.3227	20.5582	19.1144	17.7386	22.6410	38.8175	38.7110	38.9987	25.0577
	(0.319)	(0.420)	(0.273)	(0.545)	(0.838)	(0.898)	(0.930)	(0.856)	(0.515)	(0.565)	(0.549)	(0.860)
$P \nrightarrow GDP$	9.4290	9.9870	12.1148	15.1682	14.2883	21.2701	29.0528	47.5841	66.5988	59.2137	71.5165	67.1851
	(0.988)	(0.994)	(0.974)	(0.947)	(0.970)	(0.868)	(0.708)	(0.363)	(0.162)	(0.256)	(0.165)	(0.231)
$GDP \nrightarrow P$	29.7642	37.0095	33.4676	42.2190	31.1573	52.3757	51.9567	40.3790	31.6598	54.0684	65.8284	53.0823
	(0.521)	(0.351)	(0.470)	(0.328)	(0.605)	(0.238)	(0.226)	(0.459)	(0.675)	(0.244)	(0.215)	(0.342)

Table 4
Summary of causality relations at various horizons for series in first difference

h	1	2	3	4	5	6	7	8	9	10	11	12
$NBR \nrightarrow r$	⋆⋆											
$r \nrightarrow NBR$												
$NBR \nrightarrow P$	⋆⋆	⋆⋆	⋆⋆				⋆					
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$												⋆
$r \nrightarrow P$												
$P \nrightarrow r$												
$r \nrightarrow GDP$			⋆	⋆	⋆	⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆
$GDP \nrightarrow r$	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆							
$P \nrightarrow GDP$												
$GDP \nrightarrow P$						⋆		⋆	⋆			
h	13	14	15	16	17	18	19	20	21	22	23	24
$NBR \nrightarrow r$												
$r \nrightarrow NBR$												
$NBR \nrightarrow P$												
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$				⋆								
$r \nrightarrow P$												
$P \nrightarrow r$												
$r \nrightarrow GDP$	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆⋆	⋆	⋆				
$GDP \nrightarrow r$												
$P \nrightarrow GDP$												
$GDP \nrightarrow P$												

Note: The symbols ⋆ and ⋆⋆ indicate rejection of the non-causality hypothesis at the 10% and 5% levels, respectively.

Now that we have shown that our procedure does have power we present causality tests at horizon one to 24 for every pair of variables in Tables 2 and 3. For every horizon we have 12 causality tests and we group them by pairs. When we say that a given variable cause or does not cause another, it should be understood that we mean the growth rate of the variables. The p-values are computed by taking $N = 999$. Table 4 summarize the results by presenting the significant results at the 5% and 10% levels.

The first thing to notice is that we have significant causality results at short horizons for some pairs of variables while we have it at longer horizons for other pairs. This is an interesting illustration of the concept of causality at horizon h of Dufour and Renault (1998).

The instrument of the central bank, the nonborrowed reserves, cause the federal funds rate at horizon one, the prices at horizon 1, 2, 3 and 9 (10% level). It does not cause the other two variables at any horizon and except the GDP at horizon 12 and

Table 5
Causality tests and simulated p-values for extended autoregressions at the horizons 1–12

h	1	2	3	4	5	6	7	8	9	10	11	12
$NBR \nrightarrow r$	37.4523	24.0148	24.9365	22.9316	25.4508	26.6763	19.5377	19.3805	21.9278	19.8541	21.2947	19.5569
	(0.051)	(0.337)	(0.309)	(0.410)	(0.333)	(0.333)	(0.660)	(0.688)	(0.609)	(0.716)	(0.710)	(0.795)
$r \nrightarrow NBR$	25.9185	17.7977	16.6185	17.3820	19.6425	20.2032	39.8496	44.0172	43.9928	39.4217	36.7802	35.0883
	(0.281)	(0.650)	(0.747)	(0.759)	(0.673)	(0.676)	(0.129)	(0.075)	(0.130)	(0.207)	(0.286)	(0.375)
$NBR \nrightarrow P$	50.8648	44.8472	56.5028	33.8112	36.1026	42.1714	38.7204	38.8076	35.3353	33.8049	28.6205	26.9863
	(0.004)	(0.015)	(0.007)	(0.152)	(0.124)	(0.061)	(0.152)	(0.161)	(0.259)	(0.316)	(0.498)	(0.609)
$P \nrightarrow NBR$	20.7491	17.2772	16.3891	26.1610	29.7329	18.7583	22.7821	23.1743	27.0076	24.3763	31.1869	38.3349
	(0.506)	(0.704)	(0.764)	(0.339)	(0.276)	(0.730)	(0.605)	(0.629)	(0.508)	(0.655)	(0.484)	(0.336)
$NBR \nrightarrow GDP$	27.2102	23.8072	25.9561	24.3384	14.4893	17.8928	15.9036	17.9592	30.7472	33.5333	33.7687	36.3343
	(0.224)	(0.346)	(0.317)	(0.402)	(0.837)	(0.732)	(0.821)	(0.817)	(0.368)	(0.333)	(0.355)	(0.348)
$GDP \nrightarrow NBR$	16.1322	19.4471	20.6798	19.1035	29.1229	36.1053	37.1194	40.3578	43.7835	37.7337	48.3004	52.2442
	(0.746)	(0.571)	(0.537)	(0.658)	(0.269)	(0.166)	(0.167)	(0.163)	(0.138)	(0.247)	(0.125)	(0.123)
$r \nrightarrow P$	32.5147	31.2909	25.6717	24.4449	21.7640	25.4747	27.6313	31.4581	43.2611	38.2020	30.6394	19.9687
	(0.100)	(0.128)	(0.352)	(0.390)	(0.568)	(0.397)	(0.353)	(0.311)	(0.111)	(0.212)	(0.420)	(0.812)
$P \nrightarrow r$	22.7374	15.9453	15.2001	15.1933	16.2334	15.7472	13.4196	15.2506	14.4324	15.8589	21.3637	21.9949
	(0.415)	(0.704)	(0.762)	(0.790)	(0.768)	(0.830)	(0.905)	(0.859)	(0.909)	(0.887)	(0.749)	(0.731)
$r \nrightarrow GDP$	27.0435	29.5913	37.5271	35.7130	34.9901	35.1715	79.9402	92.6009	94.9068	107.6638	108.0581	138.0570
	(0.244)	(0.158)	(0.061)	(0.094)	(0.117)	(0.164)	(0.002)	(0.001)	(0.004)	(0.001)	(0.003)	(0.001)
$GDP \nrightarrow r$	41.8475	41.9449	44.7597	38.0358	30.4776	28.1840	30.5867	27.0745	26.5876	39.9502	39.8618	33.4855
	(0.032)	(0.019)	(0.019)	(0.060)	(0.209)	(0.286)	(0.250)	(0.369)	(0.431)	(0.157)	(0.181)	(0.347)
$P \nrightarrow GDP$	23.7424	26.7148	24.6605	24.6507	23.3233	19.8483	20.3581	28.9318	29.0384	27.2509	26.3318	22.6991
	(0.368)	(0.237)	(0.342)	(0.373)	(0.479)	(0.668)	(0.666)	(0.376)	(0.427)	(0.505)	(0.558)	(0.740)
$GDP \nrightarrow P$	25.1264	24.1941	25.5683	19.2127	37.5984	38.0318	37.6254	45.2219	45.2458	36.9912	23.1687	22.9934
	(0.278)	(0.358)	(0.350)	(0.639)	(0.108)	(0.110)	(0.153)	(0.088)	(0.092)	(0.237)	(0.647)	(0.698)

Table 6
Causality tests and simulated p-values for extended autoregressions at the horizons 13–24

h	13	14	15	16	17	18	19	20	21	22	23	24
$NBR \nrightarrow r$	19.5845	30.2847	17.4893	24.9745	21.2814	27.4800	36.9567	27.8970	59.3731	34.9153	27.3241	31.9143
	(0.814)	(0.513)	(0.901)	(0.750)	(0.858)	(0.706)	(0.537)	(0.770)	(0.236)	(0.640)	(0.832)	(0.771)
$r \nrightarrow NBR$	41.7360	47.8501	30.9613	29.1809	35.7654	22.9966	29.0869	29.1396	38.0675	52.5512	29.6133	25.4872
	(0.282)	(0.212)	(0.592)	(0.692)	(0.540)	(0.847)	(0.757)	(0.766)	(0.665)	(0.401)	(0.838)	(0.917)
$NBR \nrightarrow P$	36.7787	37.7357	33.4273	35.3241	20.6330	23.6911	44.5735	55.5420	53.7340	68.0270	52.3332	47.5614
	(0.359)	(0.366)	(0.512)	(0.455)	(0.860)	(0.825)	(0.379)	(0.249)	(0.290)	(0.165)	(0.363)	(0.481)
$P \nrightarrow NBR$	29.5049	39.2076	18.0831	30.6486	39.5517	34.8363	39.9608	43.6563	40.6713	44.0254	61.6914	62.9346
	(0.582)	(0.401)	(0.920)	(0.671)	(0.441)	(0.606)	(0.535)	(0.471)	(0.551)	(0.553)	(0.296)	(0.286)
$NBR \nrightarrow GDP$	30.9525	22.0737	30.1165	22.7429	17.2546	22.2686	28.6752	31.8817	39.5031	53.6466	58.8413	79.0569
	(0.501)	(0.822)	(0.599)	(0.793)	(0.937)	(0.863)	(0.749)	(0.717)	(0.591)	(0.367)	(0.327)	(0.153)
$GDP \nrightarrow NBR$	46.4538	38.0424	64.2269	60.8792	57.5798	57.2237	42.0851	43.1683	43.4379	41.7141	39.8292	41.7123
	(0.186)	(0.347)	(0.071)	(0.125)	(0.169)	(0.205)	(0.453)	(0.491)	(0.501)	(0.568)	(0.631)	(0.632)
$r \nrightarrow P$	30.7883	37.3585	30.4331	35.8788	26.9960	39.2961	44.7334	46.6740	56.9680	53.1830	67.3818	61.2522
	(0.503)	(0.364)	(0.566)	(0.448)	(0.712)	(0.429)	(0.358)	(0.352)	(0.223)	(0.310)	(0.182)	(0.241)
$P \nrightarrow r$	25.3085	33.1027	28.7362	33.5758	33.8991	39.3031	29.4228	14.3095	17.8588	29.9572	25.0347	29.1903
	(0.654)	(0.450)	(0.624)	(0.546)	(0.525)	(0.451)	(0.707)	(0.984)	(0.957)	(0.764)	(0.883)	(0.825)
$r \nrightarrow GDP$	109.5052	84.9556	88.4433	80.9836	79.9549	75.1199	94.8986	65.5929	67.2913	71.6331	75.5123	67.4315
	(0.002)	(0.011)	(0.021)	(0.016)	(0.031)	(0.073)	(0.026)	(0.147)	(0.136)	(0.164)	(0.151)	(0.254)
$GDP \nrightarrow r$	34.8185	41.4218	38.2761	28.5326	22.6116	16.7992	20.8097	31.8769	38.7083	34.4663	35.6279	20.5007
	(0.340)	(0.264)	(0.318)	(0.606)	(0.809)	(0.923)	(0.866)	(0.644)	(0.519)	(0.649)	(0.637)	(0.949)
$P \nrightarrow GDP$	8.8039	8.9511	17.0182	9.7608	16.4772	19.6942	46.3240	41.7429	64.5928	60.2875	56.0315	72.2823
	(0.995)	(0.995)	(0.933)	(0.995)	(0.923)	(0.916)	(0.349)	(0.484)	(0.207)	(0.250)	(0.332)	(0.215)
$GDP \nrightarrow P$	23.8773	39.5163	34.3832	34.5734	36.8350	54.8088	54.8048	36.4102	29.4543	58.0095	58.1341	59.5283
	(0.709)	(0.331)	(0.468)	(0.488)	(0.472)	(0.212)	(0.218)	(0.557)	(0.739)	(0.254)	(0.265)	(0.286)

16 (10% level) nothing is causing it. We see that the impact of variations in the nonborrowed reserves is over a very short term. Another variable, the GDP, is also causing the federal funds rates over short horizons (one to five months).

An interesting result is the causality from the federal funds rate to the GDP. Over the first few months the funds rate does not cause GDP, but from horizon 3 (up to 20) we do find significant causality. This result can easily be explained by, e.g. the theory of investment. Notice that we have the following indirect causality. Nonborrowed reserves do not cause GDP directly over any horizon, but they cause the federal funds rate which in turn causes GDP. Concerning the observation that there are very few causality results for long horizons, this may reflect the fact that, for stationary processes, the coefficients of prediction formulas converge to zero as the forecast horizon increases.

Using the results of Proposition 4.5 in Dufour and Renault (1998), we know that for this example the highest horizon that we have to consider is 33 since we have a

Table 7
Summary of causality relations at various horizons for series in first difference with extended autoregressions

h	1	2	3	4	5	6	7	8	9	10	11	12
$NBR \nrightarrow r$	$\star$											
$r \nrightarrow NBR$								$\star$				
$NBR \nrightarrow P$	$\star\star$	$\star\star$	$\star\star$			$\star$						
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$												
$r \nrightarrow P$	$\star$											
$P \nrightarrow r$												
$r \nrightarrow GDP$			$\star$	$\star$			$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$
$GDP \nrightarrow r$	$\star\star$	$\star\star$	$\star\star$	$\star$								
$P \nrightarrow GDP$												
$GDP \nrightarrow P$								$\star$	$\star$			

h	13	14	15	16	17	18	19	20	21	22	23	24
$NBR \nrightarrow r$												
$r \nrightarrow NBR$												
$NBR \nrightarrow P$												
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$			$\star$									
$r \nrightarrow P$												
$P \nrightarrow r$												
$r \nrightarrow GDP$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star$	$\star\star$					
$GDP \nrightarrow r$												
$P \nrightarrow GDP$												
$GDP \nrightarrow P$												

Note: The symbols $\star$ and $\star\star$ indicate rejection of the non-causality hypothesis at the 10% and 5% levels, respectively.

VAR(16) with four time series. Causality tests for the horizons 25 through 33 were also computed but are not reported. Some p-values smaller or equal to 10% are scattered over horizons 30–33 but no discernible pattern emerges.

We next consider extended autoregressions to illustrate the results of Section 5. To cover the possibility that the first difference of the logarithm of the four series may not be stationary, we ran extended autoregressions on the series analyzed. Since we used a VAR(16) with non-zero mean for the first difference of the series a VAR(17), i.e. $d = 1$, with a non-zero mean was fitted. The Monte Carlo samples with $N = 999$ are drawn in the same way as before except that the constraints on the VAR parameters at horizon h is $\hat{\pi}_{jik}^{(h)} = 0$ for $k = 1, \ldots, p$ and not $k = 1, \ldots, p + d$.

Results of the extended autoregressions are presented in Table 5 (horizons 1–12) and 6 (horizons 13–24). Table 7 summarize these results by presenting the significant results at the 5% and 10% level. These results are very similar to the previous ones over all the horizons and variable every pairs. A few causality tests are not significant anymore ($GDP \nrightarrow r$ at horizon 5, $r \nrightarrow GDP$ at horizons 5 and 6) and some causality relations are now significant ($r \nrightarrow P$ at horizon one) but we broadly have the same causality patterns.

7. Conclusion

In this paper, we have proposed a simple linear approach to the problem of testing non-causality hypotheses at various horizons in finite-order vector autoregressive models. The methods described allow for both stationary (or trend-stationary) processes and possibly integrated processes (which may involve unspecified cointegrating relationships), as long as an upper bound is set on the order of integration. Further, we have shown that these can be easily implemented in the context of a four-variable macroeconomic model of the US economy.

Several issues and extensions of interest warrant further study. The methods we have proposed were, on purpose, designed to be relatively simple to implement. This may, of course, involve efficiency losses and leave room for improvement. For example, it seems quite plausible that more efficient tests may be obtained by testing directly the nonlinear causality conditions described in Dufour and Renault (1998) from the parameter estimates of the VAR model. However, such procedures will involve difficult distributional problems and may not be as user-friendly as the procedures described here. Similarly, in nonstationary time series, information about integration order and the cointegrating relationships may yield more powerful procedures, although at the cost of complexity. These issues are the topics of on-going research.

Another limitation comes from the fact we consider VAR models with a known finite order. We should however note that the asymptotic distributional results established in this paper continue to hold as long as the order p of the model is selected according to a consistent order selection rule [see Dufour et al. (1994), Pötscher (1991)]. So this is not an important restriction. Other problems of interest would consist in deriving similar tests applicable in the context of VARMA or

VARIMA models, as well as more general infinite-order vector autoregressive models, using finite-order VAR approximations based on data-dependent truncation rules [such as those used by Lütkepohl and Poskitt (1996) and Lütkepohl and Saikkonen (1997)]. These problems are also the topics of on-going research.

Acknowledgements

This work was supported by the Canada Research Chair Program (Chair in Econometrics, Université de Montréal), the Alexander-von-Humboldt Foundation (Germany), the Institut de Finance mathématique de Montréal (IFM2), the Canadian Network of Centres of Excellence [program on *Mathematics of Information Technology and Complex Systems* (MITACS)], the Canada Council for the Arts (Killam Fellowship), the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, the Fonds de recherche sur la société et la culture (Québec), and the Fonds de recherche sur la nature et les technologies (Québec). The authors thank Jörg Breitung, Helmut Lütkepohl, Peter Schotman, participants at the EC2 Meeting on Casuality and Exogeneity in Louvain-la-Neuve (December 2001), two anonymous referees, and the Editors Luc Bauwens, Peter Boswijk and Jean-Pierre Urbain for several useful comments. This paper is a revised version of Dufour and Renault (1995).

References

Andrews, D.W.K., 1991. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica 59, 817–858.

Andrews, D.W.K., 1993. Tests for parameter instability and structural change with unknown change point. Econometrica 61, 821–856.

Andrews, D.W.K., Monahan, J.C., 1992. An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator. Econometrica 60, 953–966.

Bernanke, B.S., Mihov, I., 1998. Measuring monetary policy. The Quarterly Journal of Economics 113 (3), 869–902.

Boudjellaba, H., Dufour, J.-M., Roy, R., 1992. Testing causality between two vectors in multivariate ARMA models. Journal of the American Statistical Association 87, 1082–1090.

Choi, I., 1993. Asymptotic normality of the least-squares estimates for higher order autoregressive integrated processes with some applications. Econometric Theory 9, 263–282.

Cribari-Neto, F., Ferrari, S.L.P., Cordeiro, G.M., 2000. Improved heteroskedasticity-consistent covariance matrix estimators. Biometrika 87 (4), 907–918.

Cushing, M.J., McGarvey, M.G., 1999. Covariance matrix estimation. In: Mátyás, L. (Ed.), Generalized Method of Moments Estimation. Cambridge University Press, Cambridge, UK, pp. 63–95 (Chapter 3).

Den Haan, W.J., Levin, A., 1997. A practitioner's guide to robust covariance matrix estimation. In: Maddala, G.S., Rao, C.R. (Eds.), Handbook of Statistics, Robust Inference, vol. 15. North-Holland, Amsterdam (Chapter 15).

Doan, T., Litterman, R., 1983. RATS User's Manual. VAR Econometrics, Minneapolis.

Dolado, J.J., Lütkepohl, H., 1996. Making Wald tests work for cointegrated VAR systems. Econometric Reviews 15, 369–386.

Doornik, J.A., 1999. Object-Oriented Matrix Programming Using Ox, third ed. Timberlake Consultants Press and Oxford, Oxford, UK, www.nuff.ox.ac.uk/Users/Doornik.
Dufour, J.-M., 2002. Monte Carlo tests with nuisance parameters: a general approach to finite-sample inference and nonstandard asymptotics in econometrics. Journal of Econometrics, forthcoming.
Dufour, J.M., Khalaf, L., 2002. Simulation based finite and large sample tests in multivariate regressions. Journal of Econometrics 111 (2), 303–322.
Dufour, J.-M., Renault, E., 1995. Short-run and long-run causality in time series: inference. Technical Report, Centre de recherche et développement en économique, Université de Montréal, and Institut d'Economie Industrielle, Université des Sciences Sociales de Toulouse.
Dufour, J.-M., Renault, E., 1998. Short-run and long-run causality in time series: theory. Econometrica 66, 1099–1125.
Dufour, J.-M., Ghysels, E., Hall, A., 1994. Generalized predictive tests and structural change analysis in econometrics. International Economic Review 35, 199–229.
Dufour, J.-M., Pelletier, D., Renault, É., 2003. Short run and long run causality in time series: inference. Technical Report, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.
Gallant, A.R., 1987. Nonlinear Statistical Models. Wiley, New York.
Geweke, J., 1984. Inference and causality in economic time series. In: Griliches, Z., Intrilligator, M.D. (Eds.), Handbook of Econometrics, vol. 2. North-Holland, Amsterdam, pp. 1102–1144.
Giles, J.A., 2002. Time series analysis testing for two-step Granger noncausality in trivariate VAR models. In: Ullah, A., Wan, A.K., Chaturvedi, A. (Eds.), Handbook of Applied Econometrics and Statistical Inference. Marcel Dekker, New York, pp. 371–399 (Chapter 18).
Gouriéroux, C., Monfort, A., 1997. Time Series and Dynamic Models. Cambridge University Press, Cambridge, UK.
Granger, C.W.J., 1969. Investigating causal relations by econometric models and cross-spectral methods. Econometrica 37, 424–459.
Hansen, B.E., 1992. Consistent covariance matrix estimation for dependent heterogeneous processes. Econometrica 60, 967–972.
Kilian, L., 1998a. Confidence intervals for impulse responses under departures from normality. Econometric Reviews 17 (1), 1–29.
Kilian, L., 1998b. Small-sample confidence intervals for impulse response functions. Review of Economics and Statistics 80, 218–230.
Kurozumi, E., Yamamoto, T., 2000. Modified lag augmented vector autoregressions. Econometric Reviews 19, 207–231.
Lütkepohl, H., 1991. Introduction to Multiple Time Series Analysis. Springer, Berlin.
Lütkepohl, H., 1993. Testing for causation between two variables in higher dimensional VAR models. In: Schneeweiss, H., Zimmermann, K. (Eds.), Studies in Applied Econometrics. Springer, Heidelberg.
Lütkepohl, H., Burda, M.M., 1997. Modified Wald tests under nonregular conditions. Journal of Econometrics 78, 315–332.
Lütkepohl, H., Poskitt, D.S., 1996. Testing for causation using infinite order vector autoregressive processes. Econometric Theory 12, 61–87.
Lütkepohl, H., Saikkonen, P., 1997. Impulse response analysis in infinite order cointegrated vector autoregressive processes. Journal of Econometrics 81, 127–157.
McQuarrie, A.D.R., Tsai, C.-L., 1998. Regression and Time Series Model Selection. World Scientific, Singapore.
Newbold, P., 1982. Causality testing in economics. In: Anderson, O.D. (Ed.), Time Series Analysis: Theory and Practices, vol. 1. North-Holland, Amsterdam.
Newey, W.K., McFadden, D., 1994. Large sample estimation and hypothesis testing. In: Engle, R.F., McFadden, D.L. (Eds.), Handbook of Econometrics, vol. IV. North-Holland, Amsterdam, pp. 2111–2245 (Chapter 36).
Newey, W.K., West, K.D., 1987. Hypothesis testing with efficient method of moments estimators. International Economic Review 28, 777–787.

Paparoditis, E., 1996. Bootstrapping autoregressive and moving average parameter estimates of infinite order vector autoregressive processes. Journal of Multivariate Analysis 57, 277–296.

Paparoditis, E., Streitberg, B., 1991. Order identification statistics in stationary autoregressive moving-average models: vector autocorrelations and the bootstrap. Journal of Time Series Analysis 13 (5), 415–434.

Park, J.Y., Phillips, P.C.B., 1989. Statistical inference in regressions with integrated processes: Part 2. Econometric Theory 5, 95–131.

Pierce, D.A., Haugh, L.D., 1977. Causality in temporal systems: characterizations and survey. Journal of Econometrics 5, 265–293.

Pötscher, B., 1991. Efffects of model selection on inference. Econometric Theory 7, 163–185.

Renault, E., Sekkat, K., Szafarz, A., 1998. Testing for spurious causality in exchange rates. Journal of Empirical Finance 5, 47–66.

Sims, C., 1980. Macroeconomics and reality. Econometrica 48, 1–48.

Sims, C.A., Stock, J.H., Watson, M.W., 1990. Inference in linear time series models with some unit roots. Econometrica 58, 113–144.

Tiao, G.C., Box, G.E.P., 1981. Modeling multiple time series with applications. Journal of the American Statistical Association 76, 802–816.

Toda, H.Y., Phillips, P.C.B., 1993. Vector autoregressions and causality. Econometrica 61, 1367–1393.

Toda, H.Y., Phillips, P.C.B., 1994. Vector autoregression and causality: a theoretical overview and simulation study. Econometric Reviews 13, 259–285.

Toda, H.Y., Yamamoto, T., 1995. Statistical inference in vector autoregressions with possibly integrated processes. Journal of Econometrics 66, 225–250.

White, H., 1999. Asymptotic Theory for Econometricians, revised edition. Academic Press, San Diego, Florida.

Wiener, N., 1956. The theory of prediction. In: Beckenback, E.F. (Ed.), The Theory of Prediction. McGraw-Hill, New York (Chapter 8).

Wooldridge, J.M., 1989. A computationally simple heteroskedasticity and serial correlation robust standard error for the linear regression model. Economics Letters 31, 239–243.

Yamada, H., Toda, H.Y., 1998. Inference in possibly integrated vector autoregressive models: some finite sample evidence. Journal of Econometrics 86, 55–95.

Yamamoto, T., 1996. A simple approach to the statistical inference in linear time series models which may have some unit roots. Hitotsubashi Journal of Economics 37, 87–100.