Market failure, inequality and redistribution *

Jean-Marie Dufour †
McGill University

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† William Dow Professor of Economics, McGill University, Centre interuniversitaire de recherche en analyse des organisations (CIRANO), and Centre interuniversitaire de recherche en économie quantitative (CIREQ). Mailing address: Department of Economics, McGill University, Leacock Building, Room 519, 855 Sherbrooke Street West, Montréal, Québec H3A 2T7, Canada. TEL: (1) 514 398 8879; FAX: (1) 514 398 4938; e-mail: jean-marie.dufour@mcgill.ca . Web page: http://www.jeanmariedufour.com

ABSTRACT

We consider the following two questions: (1) What impact does market failure have on inequalities of income? (2) Does market failure justify redistribution?" Our answer to the first question is yes. But it can go in several directions, so there is no simple way to assess it from an equity viewpoint. Our answer to the second question is no, in the sense that policies for correcting market failures do not aim at producing a "desirable" income distribution. This follows from the fact that, by construction, market failure is a deviation from "efficiency" that does not involve any notion of a desirable distribution of welfare (or income). However, there are special cases where a "corrective measure" involving redistribution can offset a market failure, so this can provide a form of efficiency-based justification for redistribution.

Keywords: positive economics; normative economics; welfare economics; market failure; externality; taxation; social choice; public choice.

Journal of Economic Literature classification: A1, B4, D3, D6, D7, I3.

Contents

1. Introduction

In this paper, we consider the following two questions:

(A) What impact does market failure have on inequalities of income?

(B) Does market failure in this sense justify redistribution?"

To these questions, we give answers which represent the viewpoint of a mainstream economist. Namely, leaving out a number of qualifications – which will be discussed below – these can be formulated as follows.

(A) Yes, market failure has an impact on income inequality. But it can go in several directions, so there is no simple way to assess it from an equity viewpoint.

(B) The general answer to the second question is no, in the sense that policies for correcting market failures do not aim at producing a "desirable" income distribution. However, there are special cases where a "corrective measure" involving redistribution can also correct a market failure.

To explain these answers, we will need to discuss some basic notions and distinctions. These include the following points.

1. In policy analysis, it is important to keep in mind the distinction between positive economics and normative economics.

2. Normative economics can be split in two rather different styles of analysis:

   (a) "distribution-free" normative economics, which is based on separating aggregate wealth creation (*efficiency*) and distribution (*equity*), and focuses on the analysis of efficiency;

   (b) "distribution-sensitive" normative economics, where the levels of welfare (utilities) of different individuals are compared and weighted, for example through *social welfare functions*. Such functions can also take account of other society objectives (such as the environment).

3. "Distribution-free" normative economics does not require one to weigh the welfare of different individuals. An advantage of this approach is its neutrality with respect to one's views about a "desirable distribution of welfare". A shortcoming is its incompleteness: designing a "complete policy package" requires the introduction of distributional considerations as well as (eventually) other types of criteria, such as "ethical" criteria.

4. *Market failure* is a situation where an (idealized) market equilibrium model appears to generate inefficiencies. This has the following consequences.

   (a) "Market failure" is a relative notion not an absolute one, for it is defined as a deviation from an idealized model; if such a model is not specified or is not well-defined, the notion of market failure is itself ill-defined.

   (b) Efficiency and market failure analysis requires counterfactual experiments similar to those used in causality analysis.

   (c) The notion of market failure is associated in a fundamental way with the distinction between efficiency and equity issues. By construction, market failure does not involve any notion of a "desirable" distribution of welfare (or income).

5. In view of the above distinctions, it is possible to distinguish efficiency-enhancing policies and redistribution policies. Given redistribution policies (which may take the form of a comprehensive income security scheme, a negative income tax, etc.), equity issues can be treated through such policies.

6. We conclude that market failure does not justify redistribution, in the sense that policies for correcting market failures do not aim at producing a "desirable" income distribution. However, the "neutrality" of efficiency analysis does not mean that distribution (equity, inequality) or other ethical considerations are deemed to be unimportant for economic policy.

7. As a qualification to the efficiency/equity dichotomy, we also point out that measures aimed at "correcting" market failures (such as taxes and subsidies) typically involve some redistribution, hence providing a form of "justification" for redistribution. For example, redistribution may be viewed as a way of increasing "social consensus", which could alleviate violence in society and foster cooperation. However, when they are viewed in the broader context of "distribution-sensitive" normative economics, such distributive effects can be mediated and cancelled by redistribution policies.

Further related issues on which we shall comment include the following.

1. What are the main limitations on the separation of efficiency and distribution problems? Are they cases where the separation of efficiency and distributional issues is very difficult or infeasible? What happens when redistribution mechanisms (compensating payments) are not neutral and affect aggregate wealth? Can alternative "distribution-free" normative economic methods be developed in such cases?

2. What are the main difficulties associated with the introduction of distributional and ethical considerations in normative economics?

In section 2, we discuss the distinctions between positive and normative economics, distribution-free and distribution-sensitive normative economics. In section 3, we formulate our answers to the questions considered. Section 4 discusses a number of qualifications and related issues. We conclude in section 5.

2. Positive and normative economics

To understand the relation between market failure and redistribution, it is important to remember some basic distinctions: between positive and normative economics, as well as between different forms of normative economics.

Positive economics aims at describing, explaining and predicting "economic phenomena", such as the prices and quantities of goods and services sold in various markets, income, wealth, etc. In the view of many economists, positive economics makes economics a *scientific* discipline. In particular, the latter consists of two main types of activities: abstract theory construction, and empirical analysis. Abstract economic theory usually takes the form of models formulated in a mathematical language, where assumptions are explicitly specified and consequences are derived in a formal way. Coherence and the search for widely applicable assumptions play a major role in economic theory. Empirical analysis involves both the search for statistical regularities and the estimation and testing of economic models derived from economic theory (for example, through the use of econometric methods). The interaction between economic theory and empirical data is a central feature of modern economics. We think it is fair to say that the majority of research in economics involves empirical analysis and the assessment of theory with data.

Normative economics aims at providing instruments for comparing economic outcomes (such as policy outcomes) in a way that can be useful to decision making. This requires the expression of tastes and value judgments. For this reason, it is not usually viewed as part of economics as a *science*. However, normative economics provides a framework for a highly rational form of decision making and may require elaborate calculations. The possibility of using normative economics for *policy analysis* certainly constitutes one of the main reasons for the influence and the social importance of economics as a discipline.

Despite this difference, there is a close relationship between positive and normative economics, first through the dependence of normative economics on the findings of positive economics, but also through its general outlook on rational decision making. A basic claim of microeconomic theory – the fundamental field of positive economics – is that human behavior can be explained by preferences which provide a partial ordering of alternative possible choices (such as good baskets), without the need to introduce "cardinal" measures of utility [Hicks (1939*b*)]. In particular, some choices may be equivalent, so they cannot be ranked in a strict sense: classes of equivalent choices constitute "indifference curves".

The fact that preferences do not yield a complete ordering of possible choices does not preclude them from playing a central role in decision modeling: once a reasonably well-behaved constraint is added, such as a budget constraint or a concave production possibility frontier (which define "feasible" choices), a much reduced (typically unique) "optimal choice" can be derived. In such problems, a feasible choice to which another choice is strictly preferable can be deemed "non-admissible". Indeed, this situation is a general feature of "rational decision making".[1] The problem of hypothesis testing constitutes another classical example where no unique ranking between alternative decision rules is available, because different types of risk trade off with each other (the type I and II error risks); this situation has led to the Neyman-Pearson approach to hypothesis testing [for discussion, see Lehmann (1986) and Dufour (2000, 2001, 2003)].

On the other hand, ordinal utility makes interpersonal comparisons difficult and largely arbitrary. For this reason, *economists remain reluctant to make interpersonal comparisons* [Kaldor (1939), Hicks (1939*a*)]. Against this background, assessing economic outcomes requires taking into account the welfare of many individuals who may be differently affected by an economic situation (or by a policy). Since interpersonal comparisons are difficult, this has led to a *dual approach* to normative economics, which can be called *distribution-free* normative economics and *distribution-sensitive* normative economics.

2.1. Distribution-free normative economics

Distribution-free normative economics is based on the following ideas.

1. Resource allocations are ranked following the Pareto criterion [Pareto (1909)]. According to this criterion, a resource allocation is inefficient if it is possible to improve the welfare of at least some individuals while not lowering the welfare of the others. Otherwise, it is deemed to be efficient (in the sense of Pareto). Correspondingly, a policy is Pareto improving if it allows some agents to see their welfare improved, while losers can be compensated by a redistributive scheme. In other words, a Pareto improving policy makes the size of the "pie" larger.
   It is important to note that the Pareto ranking is only *partial* (by far not a complete one), like preferences in the basic consumer model.[2] Following the language of decision theory, it defines *admissible* and *inadmissible* allocations: under quite general assumptions, the search for an "optimal" allocation can be reduced to this potentially much reduced set. As pointed out above, the incompleteness of many rankings is a pervasive feature of decision theory: relatively uncontroversial rankings must usually be combined with more "subjective" – hence controversial – criteria in order to produce unique decisions.

[1] On the role of admissibility in decision theory, see Wald (1950) and Berger (1997)].

[2] For an example of the incompleteness of Pareto rankings in welfare analysis, see Samuelson (1950).

2. A *market failure* is a situation where the market equilibrium produces a Pareto inefficient allocation. Classical examples with respect to a perfectly competitive equilibrium include: monopoly and cartels, externalities – which may be positive (scientific knowledge) or negative (pollution) – publics goods, imperfect information, etc. Such a characterization may depend crucially on specific features of the model used. For example, what appears to be a "market failure" or a "market inefficiency" in the context of a perfect information model (where information is free) may vanish once information is represented as a costly commodity along with other commodities.

3. The concept of Pareto ranking suggests to compare resource allocations A and B by checking whether moving from A to B allows the gainers to *compensate* the losers: if this is the case, moving from A to B produces an *efficiency gain*: the "pie" to be shared has become larger.

4. Such features can be analyzed without resorting to interpersonal comparisons. Except for the assumption that more utility is preferable to less, all the analysis is based on the "scientific" techniques of positive economics.

5. In efficiency analysis, distribution issues are "bracketed" to focus on aggregate wealth. Issues related to production (efficiency) are separated from distribution, a methodology which has a long tradition in economics [see, for example, Pigou (1932), Kaldor (1939), and Hicks (1939*a*)]. Distribution-free normative economics can be viewed as a way of ranking economic outcomes under minimal "ethical assumptions", so that issues depend mostly on positive economics assumptions and results. *This does not mean that distribution (equity, inequality) or other ethical considerations are deemed to be unimportant.*

6. Many techniques used in welfare analysis are based on such ideas. For example, notions like consumer surplus, compensating variations and equivalent variations can be interpreted as techniques for measuring appropriate compensations associated with different outcomes or policies [see Just, Hueth and Schmitz (1980)]. When costs and benefits occur at different times and may be affected by uncertainty (*risk*), appropriate discount rates must be derived. These instruments play a central role in *cost-benefit analysis* [see Harberger (1976), Mishan (1971), Nas (1996)].

7. Compensated moves constitute *counterfactual simulation experiments*, quite similar to those used to perform *causal analysis* in economic and statistical models.

The theoretical foundations and the analysis of economic outcomes from a distribution-free normative viewpoint have been the subject of a considerable literature. For reviews, the reader may consult Ng (1980), Just et al. (1980) and the relevant essays in Hausman (2008, Part Three).

2.2. Distribution-sensitive normative economics

Efficiency analyses are not sufficient for government and political decision making. A final assessment usually requires taking into account distribution issues, so the welfares of different individuals (groups) must be compared and weighted, and possibly other "ethical" criteria [for a general discussion, see Hausman and McPherson (2006)].

The notion of social welfare function [Bergson (1938)] provides a systematic way of doing this. Distributional weights can be included in traditional cost-benefit analysis [for some discussion, see Harberger (1978) and Ng (1980, Appendix 9A)]. Under appropriate assumptions, using such a function leads one to pick a unique allocation among the Pareto optimal ones. But this may be too restrictive. Other approaches consist in developing criteria for deciding that certain allocations are not acceptable from a distribution viewpoint, such as allocations which allow for extreme poverty (which leads to policies aimed at satisfying *basic needs*).

From classical results in social choice theory [e.g., Arrow (1951), Arrow, Sen and Suzumura (2002)], we know that aggregating individual preferences can be a daunting, if not impossible, exercise. Formulating a social welfare function boils down to expressing preferences on the distribution of welfare in relation with other values (e.g., individual freedom), possibly on the basis of ethical and religious arguments. Differences of opinion on distributional issues depend crucially on attitudes towards economic inequality (different aversions to inequality), risk, individual freedom, the role of the state, etc. So, not surprisingly, getting different people to agree on some welfare function stand to be highly controversial if not impossible. This may motivate many economists to stick to the relatively narrow distribution-free approach.

3. Answers to the questions

Given the above discussion, our answers to the two questions formulated at the beginning are the following.

3.1. Impact of market failure on income inequality

What impact does market failure have on inequalities of income? The answer to this question is almost trivial and does not require much elaboration.

Yes, market failure has an impact on income inequality. Market failures obviously involve welfare redistribution between economic agents, for example from consumers to the holder of a monopoly (monopolistic firm, trade union), from the victims of pollution to the polluter, from the source of a positive externality (common knowledge, for example) to the beneficiaries, etc. In general, those whose lose most from a market failure may be rich or poor. Correspondingly, measures which aim at correcting market failures also have redistributive effects.

3.2. Market failure and rationale for redistribution

Does market failure justify redistribution? The discussion in section 2 shows that the notion of market failure is associated in a fundamental way with the distinction between efficiency and equity issues. By its very definition, market failure analysis involves the identification of situations where more wealth could be created while keeping its distribution constant. By construction, it is meant to abstract oneself from distributive issues. This entails that *"market" failure cannot "justify" redistribution*, in the sense that policies for correcting market failures do not aim at producing a "desirable" income distribution.

There are cases where correcting a market failure can be done by a form of redistribution, such as forcing a polluter to compensate the victims through a compensating payment (Pigou tax). In many cases, however, the group of the victims is too wide or ill-defined to allow for that. Does market failure justify redistribution?

4. Discussion

Due to the emphasis on efficiency, the problems associated with inequality and redistribution may be neglected (although certainly not ignored) in economic research. Economics cannot and does not try to have the final say on that. But it can provide useful information on the consequences of alternative redistribution policies. Important related issues concern both distribution-free and distribution-sensitive analyses.

As a first caveat to the notion that efficiency analysis is "neutral" to redistribution, it is important to note that measures that aim at correcting market failures (e.g., taxes, subsidies) almost always have distributive effects. Such effects can, however, be cancelled through specifically redistributive policies. Furthermore, it is possible to argue that redistribution may help increase "social consensus", hence potentially reducing "noncooperative behaviour" such as rebellions or criminality. If such behaviors are interpreted as negative externalities, then this could provide a direct "justification" for redistribution based on an efficiency argument. However, in the broader context of distribution-sensitive normative economics, such distributive effects can be mediated and cancelled by redistribution policies. Indeed, the very idea of "paying" people to refrain from aggression may be controversial from an ethical viewpoint.

In our view, a significant limitation of the traditional separation between efficiency and distribution problems lies in restrictions on carrying out compensating transfers. In practice, transfers are not costless and may be difficult to perform for various reasons (technical, political, etc.). A state apparatus with the ability to tax citizens is usually needed to make transfers between the members of society, whether such transfers are monetary or in-kind. What are the costs (eventually, efficiency costs) of taxation schemes needed to finance redistribution? Can a general redistribution scheme (such as a negative income tax, or some improvement) fulfill the task of redistributing economic well-being in any desired way?

This raises a more technical question: is it possible to modify traditional efficiency analysis to allow for non-neutral redistribution? In principle, nothing precludes one from taking such difficulties into account. In particular, this involves the addition of restrictions affecting the transfer process to the usual analysis, and second-best techniques may be applied [Lipsey and Lancaster (1956) and Ng (1980, Chapter 9)]. These complications have received relatively little attention and may be worth further research.

These difficulties underscore the wisdom of separating efficiency and distribution issues in the analysis of economic policies, even though this involves limitations. However, both types of normative economics matter for the economic policy process.

If we agree that efficiency analysis should be completed with the introduction of distributional and ethical considerations, this raises equally if not much more difficult problems. Besides the obvious issue of achieving agreement on appropriate distributional and ethical criteria, the following questions should be raised.

1. What are the most appropriate measures of economic inequality: income, wealth, consumption, or something else? Indeed, income, wealth and consumption distributions may evolve quite differently. For example, income inequality has increased in North America during the last 30 years, while consumption inequality has been almost stationary [Krueger and Perri (2006), Crossley and Pendakur (2002)]. The population age structure also has an effect on measured inequality, without the welfare of individuals being affected over their life cycle. This underscores that assessing economic inequality raises difficult conceptual and statistical problems which have an incidence on the construction of social welfare functions. For further discussion of measurement and trends in inequality, see Freeman (2002), Firebaugh (2003), Gadrey and Jany-Catrice (2005) and Milanovic (2007).

2. What are the actual distributive consequences of alternative policies once all the adjustments have taken place (short-run versus long-run effects)? For example, policies that may be favorable to the poor in the short-run may have the opposite effect in the long-run.

3. The political process which leads to redistribution policies involves a competition between political and opinion entrepreneurs (political parties, religious groups, public intellectuals, etc.). What are the likely consequences of this process? These issues have been extensively studied in *public choice* theory and point to central difficulties for the design of "politically acceptable" social welfare functions [Downs (1957), Buchanan and Tullock (1962), Olson (1965), Buchanan (2003)].

These problems underscore the wisdom of separating efficiency and distribution issues in the analysis of economic policies, even though this involves limitations. However, both types of normative economics matter for the economic policy process.

5. Conclusion

We have considered above the following two questions: (1) What impact does market failure have on inequalities of income? (2) Does market failure justify redistribution?

Our answer to the first question is yes. Clearly market failure can have an impact, just like almost all economic transformation or policies. Indeed, only very specific (and artificial) transformations – such as policies accompanied by compensating income variations – can be set to be neutral from a distributive viewpoint. But the direction of the effect, for example whether income distribution becomes more equal or more unequal – is in general ambiguous.

Our answer to the second question is no, in the sense that policies for correcting market failures do not aim at producing a "desirable" income distribution. This follows from the fact that, by construction, market failure is a deviation from "efficiency" that does not involve any notion of a desirable distribution of welfare (or income). Further, it is possible to distinguish efficiency-enhancing policies and redistribution policies, so equity issues can be treated through redistribution policies.

Measures aimed at "correcting" market failures (such as taxes and subsidies) typically involve some redistribution, hence providing a form of "justification" for redistribution. For example, redistribution may be viewed as a way of increasing "social consensus". However, when viewed in the broader context of "distribution-sensitive" normative economics, such distributive effects can be mediated and cancelled by redistribution policies.

A complete evaluation of economic policies requires taking into account both efficiency and distribution criteria.

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