Identification and causality in macroeconomics and finance *

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1. Introduction

Research on issues related to identification (structural modelling) and causality.

1. Identification
1.1 Theoretical issues

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1.2 Applications

1.2.1 New Keynesian Phillips curves

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2. Causality

2.1 Causality in VARMA models

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2.3 Applications

2.3.1 Small macroeconomic models

2.3.2 Finance

2. Basic points and pitfalls

1. **Indirect relations** – Statistical methods for studying simultaneous equations and causality both aim at
   analyzing and avoiding being fooled by indirect relationships:

   (a) simultaneous indirect relations (standard simultaneous equations);

   (b) intertemporal indirect relations (Granger-Wiener causality).

2. **Relative nature of simultaneity and causality** – Statements on the presence of "simultaneity biases" and "causality" are always **relative**:

   (a) a set of conditioning variables (instruments);

   (b) an information set (Granger-Wiener causality).

3. **Consequences** of the above point include the following.

   (a) It is generally meaningless to claim that estimators of a linear structural equation ("IV regression") without specifying a set of instruments which are declared to be instruments by convention (on a priori grounds);
   changing the set of instruments typically involve changing the object of interest,

   (b) Granger-Wiener causality theoretically depends on:

      i. the model used;
      ii. the aggregation of variables;
      iii. time aggregation (observation frequency).

   Changing any of these features can lead to changes in causality structures. This should be viewed as normal.

   (c) Filtering can easily distort dynamic relations and causal relations.

   (d) The time-honoured distinction between "correlation" and "causality" can easily be misleading.

In the end, everything can be reduced to predictive relationships and/or correlations.

4. **Impulse responses and causality** – Impulse responses constitute partial – possibly misleading – representations of causality properties at various horizons.
5. **Statistical and economic significance** – In statistical analysis it is important to look at both statistical significance and "economic" significance. The importance of causal links should be assessed not just tested.
6. **Statistical inference when identification may fail** – In models involving identification difficulties, many standard statistical procedures – such a standard errors and asymptotic approximations – can be highly misleading. The problem, however, can be corrected.
7. **Statistical inference in large models –** Large-sample approximations tend to be very unreliable in systems which involve many variables and parameters (VAR models, VARMA models0.

   Simulation-based statistical procedures (Monte

Carlo tests, bootstrapping) tend be very helpful in such contexts.

As far as possible, inferences based on asymptotic distributions should be controlled or replaced by simulation-based procedures.

3. Simultaneity and indirect relations

Let

$$y_t = Y_t'\beta + u_t\,,\ t,\ \ldots\ ,\, T. \tag{3.1}$$

If β is defined as the regression coefficient of y_t on Y_t, (the best linear prediction of y_t based on Y_t), i.e.

$$P_L[y_t \,|\, Y_t] = Y_t'\beta\,, \tag{3.2}$$

then

$$C[Y_t,\, u_t] = 0 \tag{3.3}$$

and β can be estimated by least squares.
If we suppose instead that

$$\begin{aligned} y_t &= P_L[y_t \,|\, z_t] + v_{1t} = z_t'\pi_2 + v_{1t}\,, & (3.4)\\ Y_t &= P_L[Y_t \,|\, z_t] + v_{2t} \ = z_t'\Pi_2 + v_{2t}\,, & (3.5)\end{aligned}$$

we have, by construction,

$$\mathsf{C}[z_t,\, v_{1t}] = 0,\ \mathsf{C}[z_t,\, v_{2t}] = 0\,. \tag{3.6}$$

Then

$$\begin{aligned} y_t &= Y_t'\beta + u_t \\ &= (z_t'\Pi_2 + v_{2t})'\beta + u_t \\ &= z_t'\Pi_2\beta + (v_{2t}'\beta + u_t) \end{aligned}$$

$$= z_t'\pi_2 + v_{1t} \tag{3.7}$$

hence

$$\pi_2 = \Pi_2\beta\,, \tag{3.8}$$

$$v_{1t} = v_{2t}'\beta + u_t\,, \tag{3.9}$$

$$u_t = v_{1t} - v_{2t}'\beta\,, \tag{3.10}$$

$$\mathsf{C}[z_t,\, u_{1t}] = 0\,. \tag{3.11}$$

If Y_t and v_{2t} are univariate,

$$\begin{aligned} E[u_t Y_t] &= E[u_t v_{2t}] \\ &= V(v_{1t}) - \mathsf{C}[v_{1t},\, v_{2t}]\beta \neq 0\,. \end{aligned} \tag{3.12}$$

Due to the introduction of the equations (3.4) - (3.5), the structural equation (3.1) cannot be consistently estimated by least squares.

This situation depends crucially on the decision to condition on the z_t (instruments).

By changing the vector z_t, the interpretation and the value of β will change (convention).

Under the above assumptions, we can also consider the regression:

$$u_t = v_{2t}' a + e_t \,, \; \mathsf{C}[e_t,\, v_{2t}] = 0, \tag{3.13}$$

hence

$$\begin{aligned} y_t &= Y_t'\beta + u_t \\ &= Y_t'\beta + v_{2t}'a + e_t \end{aligned} \tag{3.14}$$

where, by construction,

$$\mathsf{C}[e_t,\, v_{2t}] = 0 \,, \; \mathsf{C}[e_t, Y_t] = 0 \,. \tag{3.15}$$

Equation (3.14) is a regression equation.

The problem here is that v_{2t} is not observable. If

$$v_{2t} \text{ is replaced by}$$
$$\hat{v}_{2t} = Y_t - z_t'\hat{\mathit{\Pi}}_2$$

where $\hat{\mathit{\Pi}}_2$ is the least squares estimator of $\mathit{\Pi}_2$ based on regressing Y_t on z_t, the least squares estimator of β from the approximate equation

$$y_t = Y_t'\beta + \hat{v}_{2t}'a + e_t^* \,, \; t, \,\ldots\,, T \;, \tag{3.16}$$

is the 2SLS estimator of β, while a provides information on the endogeneity of Y_t in equation (3.1).

The F test for

$$H_0 : a = 0 \tag{3.17}$$

is a variant of the Durbin-Wu-Hausman test of exogeneity.
Equation (3.16) is also equivalent to

$$y_t = \hat{Y}_t'\beta + \hat{v}_{2t}'a_* + e_t^* \,,\ t, \ldots, T \,, \tag{3.18}$$

where $\hat{Y}_t = z_t'\hat{\varPi}_2$ and $a_* = a + \beta$.
The 2SLS estimator works provided

$$\varPi_2 \text{ has full column rank.} \tag{3.19}$$

This is called the **rank condition for identification**, because β must be determined by solving the equation

$$\pi_2 = \varPi_2\beta \,, \tag{3.20}$$

If identification does not hold, equation (3.18) involves an

$$\textbf{asymptotic collinearity}. \tag{3.21}$$

This brings us to the topic of identification and weak identification.

4. Nonidentification and weak identification

In the linear structural model discussed above, the structural parameter β is determined by solving the equation

$$\pi_2 = \mathit{\Pi}_2 \beta \tag{4.22}$$

for β. The latter has a unique solution in terms of the regression coefficients π_2 and $\mathit{\Pi}_2$ if and only if

$$\mathit{\Pi}_2 \text{ has full column rank.} \tag{4.23}$$

In practice, even if the identification condition holds, it appears to be often the case that structural parameters like β are "close" not to be identifiable. This can be made more precise by saying that

- $\det(\mathit{\Pi}_2' \mathit{\Pi}_2)$ is "close to zero",
  or (equivalently)
- $\mathit{\Pi}_2' \mathit{\Pi}_2$ has one or several eigenvalues "close to zero".

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable:

Sargan (1983, Econometrica)
Phillips (1984, International Economic Review)
Phillips (1985, International Economic Review)
Gleser and Hwang (1987, Annals of Statistics)
Koschat (1987, Annals of Statistics)
Phillips (1989, Econometric Theory)
Hillier (1990, Econometrica)
Nelson and Startz (1990a, Journal of Business)
Nelson and Startz (1990b, Econometrica)
Buse (1992, Econometrica)
Maddala and Jeong (1992, Econometrica)
Choi and Phillips (1992, Journal of Econometrics)
Bound, Jaeger, and Baker (1993, NBER Discussion Paper)
Dufour and Jasiak (1993, CRDE)
Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association)

McManus, Nankervis, and Savin (1994, Journal of Econometrics)

Hall, Rudebusch, and Wilcox (1996, International Economic Review)

Dufour (1997, Econometrica)

Shea (1997, Review of Economics and Statistics)

Staiger and Stock (1997, Econometrica)

Wang and Zivot (1998, Econometrica)

Zivot, Startz, and Nelson (1998, International Economic Review)

Startz, Nelson, and Zivot (1999, International Economic Review)

Perron (1999)

Stock and Wright (2000, Econometrica)

Dufour and Jasiak (2001, International Economic Review)

Dufour and Taamouti (2001)

Kleibergen (2001, 2002)

Moreira (2001, 2002)

Stock and Yogo (2002)

Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)

Dufour (2003, Canadian Journal of Economics)

Dufour and Taamouti (2005, Econometrica)

Dufour and Taamouti (2006, Journal of Econometrics, forth.)

Surveys:

- Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)
- Dufour (2003, Canadian Journal of Economics)

Weak instruments have been notorious to cause serious statistical difficulties, form the viewpoints of:

1. estimation;
2. confidence interval construction;
3. testing.

Difficulties

1. Theoretical results show that the distributions of various estimators depend in a complicated way upon unknown nuisance parameters. So they are difficult to interpret.
2. When identification conditions do not hold, standard asymptotic theory for estimators and test statistics typically collapses.
3. With weak instruments,
   (a) 2SLS becomes heavily biased (in the same direction as OLS),
   (b) distribution of 2SLS is quite far the normal distribution (e.g., bimodal).

4. Standard Wald-type procedures based on asymptotic standard errors become fundamentally unreliable or very unreliable in finite samples [Dufour (1997, Econometrica)].

5. Problems were strikingly illustrated by the reconsideration by Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association) of a study on returns to education by Angrist and Krueger (1991, QJE):

   - 329000 observations;
   - replacing the instruments used by Angrist and Krueger (1991, QJE) with randomly generated instruments (totally irrelevant) produced very similar point estimates and standard errors;
   - indicates that the instruments originally used were weak.

Crucial to use finite-sample approaches to produce reliable inference.

Finite-sample approaches to inference on models involving weak identification

- Dufour (1997, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Taamouti (2005, Econometrica)
- Beaulieu, Dufour, and Khalaf (2005)
- Dufour and Valéry (2005)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)
- Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)
- Dufour, Khalaf, and Kichian (2006b)
- Dufour, Khalaf, and Kichian (2006d)

Important features

1. Procedures robust to lack of identification (or weak identification)
2. Procedures for which a finite-sample distributional theory can be supplied, at least in some reference cases
3. Limited information methods which do not require a complete formulation of the model [limited-information vs. full-information methods]
   (a) Robustness to missing instruments
   (b) Robustness to the formulation of the model for the explanatory endogenous variables

5. Weak identification and New Keynesian Phillips Curves

For basic NKPC, the issue of weak identification has been considered by several authors:

Ma (2002, Economics Letters)

Khalaf-Kichian (2004)

Mavroeidis (2004, Oxford Bulletin of Economics and Statistics)

Mavroeidis (2005, JMCB)

Yazgan-Yilmazkuday (2005, Studies in Nonlinear Dynamics and Econometrics)

Nason and Smith (2005)

Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)

Mavroeidis (2006)

1. Dufour, J.-M., L. Khalaf, and M. Kichian (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30 (9-10), 1707–1727.

Gali-Gertler (JME, 1999) model

$$
\begin{aligned}
\underbrace{\pi_t}_{\text{inflation}} &= \lambda \underbrace{s_t}_{\text{marginal costs}} + \gamma_f \boxed{E_t \pi_{t+1}} + \gamma_b \, \pi_{t-1} \\
&= \lambda s_t + \gamma_f \pi_{t+1} + \gamma_b \, \pi_{t+1} + u_{t+1}
\end{aligned}
$$

$$
\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega - \omega\theta + \omega\beta\theta}
$$

$$
\gamma_f = \frac{\beta\theta}{\theta + \omega - \omega\theta + \omega\beta\theta} \blacktriangleright \underline{\text{forward-looking}}
$$

$$
\gamma_b = \frac{\omega}{\theta + \omega - \omega\theta + \omega\beta\theta} \blacktriangleright \underline{\text{backward-looking}}
$$

$$
\beta \equiv \text{ subjective discount rate}
$$

- Identification-robust tests and CS for model parameters $(\lambda, \gamma_f, \gamma_b)$ and $(\omega,\, \theta, \beta)$ based on AR-type statistics and projection techniques.
- Rational and survey expectations studied.
- Survey expectations variants rejected.
- Model acceptable for the U.S. but not for Canada.

2. Dufour, J.-M., L. Khalaf, and M. Kichian (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Calvo-type inflation model studied by Eichenbaum and Fisher (2005) model.

3. Dufour, J.-M., L. Khalaf, and M. Kichian (2006c): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Lindé (JME, 2005) multi-equation NKPC.

6. Short-run and long-run causality

It is possible that a variable Y does not cause a variable X at horizon 1, but causes it at horizon $h > 1$ (indirect causality transmitted by an auxiliary variable Z)
[Lütkepohl (1993), Dufour and Renault (1998)].

$$\begin{pmatrix} X(t+1) \\ Y(t+1) \\ Z(t+1) \end{pmatrix} = \begin{bmatrix} 0.60 & 0.00 & 0.80 \\ 0.00 & 0.40 & 0.00 \\ 0.00 & 0.60 & 0.10 \end{bmatrix} \begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} + \begin{pmatrix} \varepsilon_X(t+1) \\ \varepsilon_Y(t+1) \\ \varepsilon_Z(t+1) \end{pmatrix}$$

$$\Rightarrow \; X(t+1) = 0.6X(t) + 0.8Z(t) + \varepsilon_X(t+1).$$

Since the coefficient of $Y(t)$ is 0, we can conclude that Y does not cause X at horizon 1 [Wiener (1956), Granger (1969, Econometrica)].

If we consider the above model at time $(t+2)$:

$$\begin{aligned} X(t+2) = \; & 0.36\, X(t) + 0.48Y(t) \\ & +0.56\, Z(t) + 0.6\varepsilon_X(t+1) \\ & +0.8\varepsilon_Z(t+1) + \varepsilon_X(t+2). \end{aligned}$$

The coefficient of $Y(t)$ is equal to 0.48, which implies that Y causes X at horizon 2. Here we are in presence of an indirect effect $(0.48 = 0.80 \times 0.60)$,

$$Y \stackrel{0.6}{\rightarrow} Z \stackrel{0.8}{\rightarrow} X$$

1. Processes:

   $\{X(t) : t \in \mathbb{Z}\}$, $\{Y(t) : t \in \mathbb{Z}\}$, $\{Z(t) : t \in \mathbb{Z}\}$

   $X(t)$ and $Y(t)$ scalar, $Z(t)$ vector.

2. Information sets:

$$\begin{aligned} \underline{\mathbf{X}}_t &= \{X(s), s \leq t\}, \\ \underline{\mathbf{Y}}_t &= \{Y(s), s \leq t\}, \\ \underline{\mathbf{Z}}_t &= \{Z(s), s \leq t\}, \\ I_t &= \underline{\mathbf{X}}_t \cup \underline{\mathbf{Y}}_t \cup \underline{\mathbf{Z}}_t. \end{aligned}$$

3. The variance of the forecast error of $X(t+h)$ based on the information set A_t, for $A_t = I_t$, $I_t - \underline{\mathbf{Y}}_t = \underline{\mathbf{X}}_t \cup \underline{\mathbf{Z}}_t$:

$$\sigma^2(X(t+h) \mid A_t).$$

   Linear prediction.

Definition 6.1 *For $h \geq 1$, we say that Y does not cause X at horizon h given all elements of I_t except the past of Y, denoted*

$$Y \underset{h}{\nrightarrow} X \mid Z$$

if

$$\sigma^2(X(t+h) \mid I_t - \underline{Y}_t) = \sigma^2(X(t+h) \mid I_t),\ \forall t \geq 0\,.$$

To be more specific, let

$$W(t) = \sum_{j=1}^{\infty} \pi_j W(t-j) + a(t) \qquad (6.1)$$

Then

$$W(t) = (X(t)', Y(t)', Z(t)')' . \qquad (6.2)$$

Then the best linear forecast of $W(t+h)$ given the history I_t of the process at time t is

$$P[W(t+h)|I_t] = \sum_{j=1}^{\infty} \pi_j^{(h)} W(t+1-j) \qquad (6.3)$$

where

$$\pi_j^{(1)} = \pi_j \ , \quad \pi_j^{(h+1)} = \pi_{j+1}^{(h)} + \pi_1^{(h)} \pi_j \ , \ h = 1, 2, \ \ldots \qquad (6.4)$$

Setting

$$\pi_j^{(h)} = \begin{bmatrix} \pi_{XXj}^{(h)} & \pi_{XYj}^{(h)} & \pi_{XZj}^{(h)} \\ \pi_{YXj}^{(h)} & \pi_{YYj}^{(h)} & \pi_{YZj}^{(h)} \\ \pi_{ZXj}^{(h)} & \pi_{ZYj}^{(h)} & \pi_{ZZj}^{(h)} \end{bmatrix} \qquad (6.5)$$

we have:

$$Y \underset{h}{\nrightarrow} X|I_{XZ} \Leftrightarrow \pi_{XYj}^{(h)} = 0\ ,\ \forall j \in \mathbb{N}\ . \tag{6.6}$$

Let us consider the moving average representation of the process (under stationarity):

$$W(t) = \sum_{j=0}^{\infty} \psi_j a(t-j)\ . \tag{6.7}$$

Then

$$\pi_1^{(h)} = \psi_h\ , \quad \forall h \geq 0\ . \tag{6.8}$$

Setting

$$\psi_h = \begin{bmatrix} \psi_{XXh} & \psi_{XYh} & \psi_{XZh} \\ \psi_{YXh} & \psi_{YYh} & \psi_{YZh} \\ \psi_{ZXh} & \psi_{ZYh} & \psi_{ZZh} \end{bmatrix}\ ,\ h \geq 0\ , \tag{6.9}$$

the condition

$$\psi_{XYh} = 0,\ \text{for } h \geq 0 \tag{6.10}$$

is neither necessary nor sufficient for $Y \underset{h}{\nrightarrow} X|I_{XZ}$.

Example of discrepancy between impulse responses and causality

Consider a trivariate process $W_t = (X_t\,, Y_t\,, Z_t)'$ with the VAR(6) representation:

$$
\begin{aligned}
W_t &= \begin{bmatrix} .30 & .05 & .40 \\ -.30 & .30 & -.40 \\ -.50 & -.10 & .30 \end{bmatrix} W_{t-1} + \begin{bmatrix} -.30 & .01 & .40 \\ .30 & .10 & .10 \\ .30 & .10 & .10 \end{bmatrix} W_{t-2} \\
&+ \begin{bmatrix} -.20 & .03525 & -.10 \\ -.50 & .30 & -.50 \\ -.20 & .20 & -.10 \end{bmatrix} W_{t-3} + \begin{bmatrix} .20 & -.1256 & -.10 \\ .30 & -.10 & -.10 \\ .10 & 0 & -.10 \end{bmatrix} W_{t-4} \\
&+ \begin{bmatrix} -.10 & -.12659625 & .50 \\ -.10 & -.10 & 0 \\ .10 & 0 & -.10 \end{bmatrix} W_{t-5} + \begin{bmatrix} .10 & .02820225 & .40 \\ .30 & .30 & .40 \\ .30 & -.20 & .30 \end{bmatrix} W_{t-6} + u_t\,.
\end{aligned}
\tag{6.11}
$$

In the above model, the coefficients $\pi_{XYj}\,,\ j = 2,\ \dots,\ 6,$ were chosen so that $\pi_{XY1}^{(h)} = 0,\ h = 2,\ \dots,\ 6$ (the latter are reported with a higher precision to make the results easily checkable). The following table gives the coefficients $\pi_{XYj}^{(h)}$, for $j = 1, 2, 3, 4, 5, 6,$ and $h = 1, 2, 3, 4, 5, 6.$

Table 6.11**:** $\pi^{(h)}_{XYj} \times 10^3$ **corresponding to** (6.11)

h╲j	1	2	3	4	5	6
1	50.0	10.0	35.3	-125.6	-126.6	28.2
2	0	83.3	-20.0	-169.3	-14.8	-56.5
3	0	37.7	-60.3	38.6	-2.7	-136.0
4	0	-58.1	33.9	71.7	-61.0	-32.9
5	0	-16.3	-26.9	-77.3	-49.4	106.8
6	0	-14.7	-45.8	-109.1	46.7	-1.4

7. Causality at different horizons in macroeconomic and financial data

Empirical studies of causality at different horizons

1. Dufour, Pelletier, and Renault (2006): tests of causality at different horizons on monetary policy data previously studied by Bernanke and Mihov (1998).
2. Dufiour and Tessier (2006): tests of causality at different horizons to study the relationship between monetary policy variables and stock prices.
3. Dufour and Taamouti (2008): causality measures at different horizons on monetary policy data previously studied by Bernanke and Mihov (1998).
4. Dufour, Garcia, and Taamouti (2008): causality measures to study the interactions between stock returns, realized volatility and implied volatility, based on high-frequency data on S&P 500 Index futures contracts.

   This sheds light on the relative merits of the leverage hypothesis and the volatility feedback.

In Dufour, Pelletier, and Renault (2006), we reconsider the data set used by Bernanke and Mihov (1998) in order to study United States monetary policy: monthly observations (January 1965 to December 1996, 384 observations) on:

1. nonborrowed reserves (NBR, also denoted w_1);
2. the federal funds rate $(r,\ w_2)$;
3. the GDP deflator $(P,\ w_3)$;
4. real GDP $(GDP,\ w_4)$.

We propose a simple to implement linear estimation method in conjunction with bootstrapping to test causality at different horizons.

8. Conclusion

1. Econometric methods can provide useful information in assessing structural models and issues related to causal links.

2. It is important to formulate clearly what we mean we speak of simultaneity problems and causal links.

3. It is important to be careful with respect to finite-sample issues. Nowadays simulation-based procedures provide efficient solutions, or at least reasonable quick fixes to many difficult econometric problems.

Table 1. Summary of causality relations at various horizons for series in first difference

h	1	2	3	4	5	6	7	8	9	10	11	12
$NBR \nrightarrow r$	$\star\star$											
$r \nrightarrow NBR$												
$NBR \nrightarrow P$	$\star\star$	$\star\star$	$\star\star$				$\star$					
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$												$\star$
$r \nrightarrow P$												
$P \nrightarrow r$												
$r \nrightarrow GDP$			$\star$	$\star$	$\star$	$\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$
$GDP \nrightarrow r$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$							
$P \nrightarrow GDP$												
$GDP \nrightarrow P$						$\star$		$\star$	$\star$			

h	13	14	15	16	17	18	19	20	21	22	23	24
$NBR \nrightarrow r$												
$r \nrightarrow NBR$												
$NBR \nrightarrow P$												
$P \nrightarrow NBR$												
$NBR \nrightarrow GDP$												
$GDP \nrightarrow NBR$				$\star$								
$r \nrightarrow P$												
$P \nrightarrow r$												
$r \nrightarrow GDP$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star\star$	$\star$	$\star$				
$GDP \nrightarrow r$												
$P \nrightarrow GDP$												
$GDP \nrightarrow P$												

Note _ The symbols $\star$ and $\star\star$ indicate rejection of the non-causality hypothesis at the 10% and 5% levels respectively.

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