## PREDICTIVE TESTS FOR STRUCTURAL CHANGE AND THE ST.LOUIS EQUATION*

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ABSTRACT

The main purpose of the paper is to illustrate the use of a dummy variable interpretation of the predictive Chow test against structural change. After describing how the predictive Chow test against structural change in linear regression models can be viewed as a test on the coefficients of a set of dummy variables, it is shown that these can provide useful additional information on the importance and timing of structural changes. Then, the approach is illustrated by applying it to a version of the St. Louis equation (in rate-of-change form) estimated over the period 1953/I-1976/IV: we detect some instability in the 1970's but find it is rather localized, being linked mainly to two quarters (1973/IV and 1975/III).


### 1. INTRODUCTION

An important way of assessing the reliability of an econometric model consists in checking whether it is stable over time [see Lucas (1976)]. Frequently this problem can be formalized as one of testing whether the coefficient vectors in two regressions (corresponding to disjoint subperiods) are equal. Namely, one considers:

$$\underline{y}_i = X_i\underline{\beta}_i + \underline{u}_i \ , \ i=1,2 \qquad (1)$$

where $\underline{y}_i$ is a $T_i\times 1$ vector of observations on a dependent variable, $X_i$ is a $T_i\times k$ non-stochastic matrix of explanatory variables, $\underline{\beta}_i$ is a vector of coefficients and $\underline{u}_i$ a $T_i\times 1$ vector of disturbances $(i=1,2)$. We assume also that $(\underline{u}_1',\underline{u}_2')' \sim N[\underline{0},\sigma^2 I_T]$, where $T = T_1+T_2$. The hypothesis to be tested is $H_0:\underline{\beta}_1 = \underline{\beta}_2$. It is customary to assume further that $\text{rank}(X_1) = k<T_1$ and to consider two distinct cases according to whether $\text{rank}(X_2)=k<T_2$ or $\text{rank}(X_2) = T_2 \leq k$. Tests known to econometricians as the «Chow tests» may then be applied. In the first case, a standard analysis-of-covariance test may be used [see Chow (1960, p. 598)]. For the second case, a predictive test was suggested by Chow (1960) and shown by him to be equivalent to a test based on the statistic:

$$F_1 = \frac{(SS_0 - SS_1)/T_2}{SS_1/(T_1-k)} \qquad (2)$$

where $SS_1$ is the residual sum of squares based on the first $T_1$ observations and $SS_0$ is the residual sum of squares based on all T observations; under the null hypothesis, this statistic follows an F distribution with $(T_2,T_1-k)$ degrees of freedom. The latter test remains applicable when $\text{rank}(X_2) = k < T_2$ and may, even in this case, have a better power against $H_0$ than the analysis-of-covariance test [see Wilson (1978)].

On the other hand, it has been stressed by Gujarati (1970a,b) that the analysis-of-covariance test can also be performed via the use of dummy variables and that the extra coefficients can provide useful additional information: although both methods yield an identical conclusion concerning the null hypothesis $H_0$, the latter has the advantage of automatically producing indications on the sources of difference between the two regressions, i.e. on which coefficients may have changed between the periods. However, when the second sample is undersized $(T_2 < k)$, the relevant explanatory-variable matrix does not have full column rank and thus Gujarati's procedure is not applicable. To the extent of our knowledge, no comparable interpretation has been given to the predictive test.

The first purpose of this note is to propose an interpretation of the predictive Chow test as a test on a set of dummy variables and to show that these can also provide revealing additional information on possible structural changes. To be more specific, the predictive Chow test indicates whether there is, among the $T_2$ observations predicted, at least one observation whose mean is inconsistent with the model of the first $T_1$ observations; nevertheless, when $T_2 \geq 2$, it does not point out which ones among the $T_2$ «extra observations» deviate most strongly from this model and thus, when $H_0$ is rejected, may be causing the rejection of $H_0$. Of course, this knowledge can be of great use in assessing the importance and determining the causes of a structural change. We show below that a dummy variable approach provides a computationally very convenient method for performing predictive tests on each individual extra observation and that this approach can be fruitfully used in analyzing structural change. The second purpose of this paper is to give an especially simple proof of the distribution of the predictive test statistic. Indeed, while the null distribution of the analysis-of-covariance test statistic can be obtained by showing it is a case of a linear hypothesis test on the coefficients a full-rank linear regression model, a similar proof has not apparently been given for the predictive Chow test and, consequently, a number of special proofs had to be devised for it [see Chow (1960), Fisher (1970) and Harvey (1976)]: we show below that the dummy variable interpretation of the same test provides a simple and natural way of making such a proof, similar to the one available for the first «Chow test».[1]

The alternative proof and interpretation of the predictive Chow test is described in section 2. Its use in performing predictive tests on individual observations is discussed in section 3. Results of an application to the St.Louis equation are reported in section 4.

### 2. ALTERNATIVE PROOF

Let us define $\underline{y} = (\underline{y}_1', \underline{y}_2')'$, $\underline{u} = (\underline{u}_1',\underline{u}_2')'$, $X = [X_1', X_2']'$ and

$$X^* = \begin{bmatrix} X_1 & 0 \\ X_2 & I_{T_2} \end{bmatrix}$$

where $I_{T_2}$ is the identity matrix of order $T_2$. We assume $\text{rank}(X_1) = k<T_1$ and $\text{rank}(X_2) = \min\{k,T_2\}$. It follows, since $\text{rank}(X_1) = k$ and $\text{rank}(I_{T_2}) = T_2$, that the $T\times(k+T_2)$ matrix X has full column rank. Then let us consider the regression:

$$\underline{y} = X^* \begin{bmatrix} \underline{\beta} \\ \underline{\gamma} \end{bmatrix} + \underline{u} \ , \qquad (4)$$



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where $\beta$ and $\gamma$ are vectors of coefficients of dimensions $k \times 1$ and $T_2 \times 1$ respectively. The null hypothesis $H_0: \beta_1 = \beta_2 \equiv \beta$ is equivalent to $\gamma = 0$, and we can thus test it by testing $\gamma = 0$ in (4). In other words, we add a dummy variable for each observation in the second regression and test whether all the coefficients of these dummy variables are zero. The standard F-test of $\gamma = 0$ is based on the statistic:

$$F' = \frac{(SS_0 - SS_1')/T_2}{SS_1'/(T-T_2-k)} \tag{5}$$

where

$$SS_0 = \min_{\beta} ||y - X\beta||^2 , \tag{6}$$

$$SS_1' = \min_{\beta,\gamma} \left[ ||y_1 - X_1\beta||^2 + ||y_2 - X_2\beta - \gamma||^2 \right], \tag{7}$$

and $||x||^2$ represents the sum of squares of the components of vector $x$. Under the null hypothesis, $F'$ follows an F distribution with $(T_2, T-T_2-k)$ degrees of freedom. Now, since we can always set $\hat{\gamma} = y_2 - X_2\hat{\beta}_1$, where $\hat{\beta}_1$ is the value of $\beta$ obtained while finding $SS_1'$, we must have:

$$SS_1' = \min_{\beta} ||y_1 - X_1\beta||^2 = SS_1;$$

hence, since $T-T_2-k = T_1-k$, we see that $F' = F_1$ and thus $F_1$ follows an F-distribution with $(T_2, T_1-k)$ degrees of freedom.

Note that this proof of the distribution of $F_1$ is valid whether $T_2 \leq k$ or $T_2 > k$, and that it would also hold if, instead of $I_{T_2}$ in $X^*$, we had used any $T_2 \times T_2$ non-singular matrix $Z$; in that case, one simply sets $\hat{\gamma} = Z^{-1}(y_2 - X_2\hat{\beta}_1)$.

## 3. PREDICTIVE TESTS ON INDIVIDUAL OBSERVATIONS

Let us now examine more closely what the coefficient vector $\gamma$ represents. If we rewrite equation (4) in the form

$$y_t = x'_t\beta + \sum_{s=T_1+1}^{T} \gamma_s D_{ts} + u_t, \quad t=1,\ldots,T , \tag{8}$$

where $x'_t = (x_{t1},\ldots,x_{tk})$ is the t-th line of the matrix X, $\gamma = (\gamma_{T_1+1},\ldots,\gamma_T)'$ and

$$D_{ts} = 1, \; t=s$$
$$= 0, \; t \neq s$$

we can see easily that

$$\gamma_s = E(y_s) - x_s'\beta , \quad s = T_1+1,\ldots,T ; \tag{9}$$

i.e. $\gamma_s$ is the deviation of the actual mean of $y_s$ from the mean predicted by the «common coefficient vector $\beta$». These deviations can be estimated, in the process of performing the predictive Chow test, by estimating equation (4) instead of the usual equation $y = X\beta + u$ . From the above proof, we can see that $\hat{\gamma} = y_2 - X_2\hat{\beta}_1$, where $\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'y_1$, and thus the covariance matrix of $\hat{\gamma}$ is $\sigma^2 V$, where

$$V = I_{T_2} + X_2(X_1'X_1)^{-1}X_2'. \tag{10}$$

If equation (4) is estimated using any standard regression package the estimate of $\sigma^2 V$ produced will be of the form $s_1^2 V$ , where $s_1^2 = SS_1/(T_1-k)$. Since $s_1^2$ is an unbiased estimate of $\sigma^2$, it is necessary that $V_1 = V$. Furthermore, standard errors and t-statistics are usually produced automatically for each coefficient $\gamma_s$; from (10), the empirical standard error of $\hat{\gamma}_s$ is $s_1 d_s$, where $d_s = [1 + x_s'(X_1'X_1)^{-1}x_s]^{\frac{1}{2}}$ , while the t-statistic associated with it is

$$t_s = (y_s - x_s'\hat{\beta}_1)/s_1 d_s , \quad s = T_1+1,\ldots,T; \tag{11}$$

under the null hypothesis $\gamma_s = 0$, $t_s$ follows a Student-t distribution with $T_1-k$ degrees of freedom. These statistics are the predictive test statistics for each observation $s=T_1+1,\ldots,T$, based on the parameter estimates obtained from the first $T_1$ observations. Their interest as diagnostic checks consists in pointing out which one of these observations deviate most strongly from the model of the first $T_1$ observations.[2]

## 4. APPLICATION

We applied the above technique to the «St. Louis equation» in rate-of-change form, suggested by Carlson (1978):

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i} + u_t$$

where $\dot{Y}_t$, $\dot{M}_t$ and $\dot{E}_t$ are the compounded annual rates of change in GNP, money stock (M1) and high-employment expenditures respectively in the United States. The sample period considered is 1953/I-1976/IV (quaterly data). This equation was originally estimated using Almon polynomials for $m_i$ and $e_i$ [fourth degree polynomials constrained to go through zero at the endpoints; see Carlson (1978, Table IV)]. However, we also estimated this equation without restrictions (see Table 1) and found that the F statistic for testing the Almon restrictions is quite high [$F_{4,85} = 2.608 > F_{.05}(4,85)$]. Consequently, we reject the Almon restrictions and we shall concentrate our analysis on the less restricted model (though the results of the stability analysis of the constrained specification will also be reported); in any case, from Table 1, we can observe that values of $\Sigma m_i$ and $\Sigma e_i$ are very similar to those obtained by Carlson and yield the same policy implications.

In order to test for structural change, we divided the sample into two subperiods: 1953/I-1969/IV and 1970/I-1976/IV. The analysis-of-covariance test statistic against structural change is then $F_{11,74} = 2.559$ while the predictive test statistic is $F_{28,57} = 1.692$; these are significant at levels as low as .0084 and .047 respectively.[3] Following the approach described above, the result of estimating the same equation over the period 1953/I-1976/IV with a dummy variable for each observation in 1970/I-1976/IV is reported in Table 2. From it, we can see that two of the prediction errors [1973/IV and 1975/III] are appreciably larger than the others and significant at levels much lower than the conventional .05 level; besides, 1970/IV, 1971/I and 1971/III show p-values (marginal significance levels) near .05 while the 23 other prediction errors appear relatively small.[4] Thus, it seems that the instability identified by the two Chow tests should be related to events which especially affected the economy in 1973/IV and 1975/III. Indeed, if one drops these two observations from the regression, the analysis-of-covariance and predictive test statistics become 1.724 and

1.242 respectively, none of which is significant at level .05 (the marginal significance levels are .085 and .244 respectively). We have thus identified two observations whose behavior is sufficient to make the mull hypothesis of stability being rejected.

Futhermore, it is interesting to observe that 1973/IV is the quarter where the Arab oil embargo and OPEC price hikes started, two very important and rather special disruptions; moreover, it coincides with the end of a period of expansion of real GNP [1971/I-1973/IV] and the beginning of the «Great Recession» of 1974/I-1975/I. Similarly, 1975/III more or less coincides with the beginning of the recovery from the same recession; besides, we may note that an important (though temporary) tax cut, including income tax reductions (part of which were a tax rebate on 1974 taxes) and an investment tax credit for businesses, was enacted during 1975/II [see Blinder (1979, pp. 150-152)], which may explain part of the underprediction phenomenon in this case. Finally, we can observe that 1970/IV is the last quarter of the recession of 1970. Therefore the St. Louis equation in the specification considered above shows signs of instability although these appear rather localized; in particular it seems least appropriate around business cycle turning points and, especially, for dealing with important supply shocks.

## 5. CONCLUDING REMARKS

Regression (4) thus provides a computationally very convenient method for obtaining direct evidence on one of the main consequences of structural instability (large prediction errors) jointly with a whole array of predictive test statistics.[5] Without the dummy variable method, one would need to perform $T_2$ extra regressions or to compute the $t_s$ statistics explicitly, which may be quite burdensome. Further, when $T_2>k$, the dummy variables considered above do not become identical with those used by Gujarati (1970a,b) and give a different type of information, relating to the timing of structural change rather than coefficients. Thus the analysis-of-covariance test and the predictive test give complementary information and it may be useful to perform both tests (with dummies) whenever possible. Finally it can be pointed out that the simple and natural parametric interpretation given above of the predictive Chow test (as a test on the parameter vector $\underline{\gamma}$) shows clearly how the predictive test is a test designed against a much wider set of alternatives than the analysis-of-covariance test (for $\underline{\beta}_1 \neq \underline{\beta}_2$ is a special case of $\gamma \neq 0$, while the converse does not hold); furthermore, this set-up makes straight-forward the construction of Bayesian posterior odds in the case $T_2 \leq k$, since all that is needed is putting a prior distribution on $\underline{\gamma}$.[6]

*Table 1*
*Unconstrained St. Louis Equation**

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i}$$

Sample period: 1953/I-1976/IV

| | | | |
|---|---|---|---|
| $m_0$ | .607 ( 3.277) | $e_0$ | .0550 ( 1.329) |
| $m_1$ | .238 ( 1.031) | $e_1$ | .104 ( 2.511) |
| $m_2$ | .022 ( .093) | $e_2$ | -.0225 (- .557) |
| $m_3$ | .631 ( 2.536) | $e_3$ | -.0276 (- .688) |
| $m_4$ | -.440 (-2.260) | $e_4$ | -.0947 (-2.418) |
| $\Sigma m_i$ | 1.059 ( 2.315) | $\Sigma e_i$ | .0141 ( .211) |
| $\alpha$ | 2.829 ( 3.507) | | |

SS = 1116.54, $R^2$ = .465, S.E. = 3.624
D.W. = 1.745, D.F. = 85

* t-statistics are given in parentheses, SS is the sum of squared residuals, $R^2$ is the coefficient of multiple determination, S.E. is the standard error of the regression, D.W. is the Durbin-Watson statistic and D.F. is the number of degrees of freedom.

*Table 2*
*Unconstrained St. Louis Equation with Dummies*

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i} + \sum_{s=1970/I}^{1976/IV} \gamma_s D_{ts}$$

Sample period: 1953/I-1976/IV

| | | | |
|---|---|---|---|
| $m_0$ | .527 ( 2.355) | $e_0$ | .0540 ( 1.044) |
| $m_1$ | .202 ( .687) | $e_1$ | .107 ( 2.008) |
| $m_2$ | .301 ( 1.010) | $e_2$ | .0285 ( .571) |
| $m_3$ | .490 ( 1.661) | $e_3$ | - .0889 (-1.831) |
| $m_4$ | - .441 (-1.962) | $e_4$ | - .166 (-3.630) |
| $\Sigma m_i$ | 1.079 ( 4.900) | $\Sigma e_i$ | - .0656 (- .097) |
| $\alpha$ | 3.262 ( 4.052) | | |
| $\gamma_s$ | | $\gamma_s$ | |
| 1970/I | -1.312 (- .383) | 1973/III | .621 ( .153) |
| 1970/II | -2.077 (- .580) | 1973/IV | 9.447 ( 2.443)** |
| 1970/III | -3.058 (- .831) | 1974/I | -6.419 (-1.861) |
| 1970/IV | -7.297 (-2.002)* | 1974/II | -1.814 (- .528) |
| 1971/I | 7.394 ( 2.027)* | 1974/III | -1.459 (- .428) |
| 1971/II | - .623 (- .165) | 1974/IV | -2.359 (- .697) |
| 1971/III | -7.238 (-1.990)* | 1975/I | -4.416 (-1.257) |
| 1971/IV | -2.073 (- .581) | 1975/II | 5.222 ( 1.391) |
| 1972/I | 2.671 ( .702) | 1975/III | 10.952 ( 2.934)** |
| 1972/II | 1.782 ( .467) | 1975/IV | 4.339 ( 1.141) |
| 1972/III | -1.830 (- .478) | 1976/I | 5.970 ( 1.611) |
| 1972/IV | - .294 (- .071) | 1976/II | 4.332 ( 1.147) |
| 1973/I | 3.512 ( .850) | 1976/III | 4.630 ( 1.184) |
| 1973/II | -3.928 (- .951) | 1976/IV | -3.407 (- .902) |

SS = 609.805, $R^2$ = .555, S.E. = 3.271,
D.W. = 1.867, D.F. = 57

* Marginal significance levels for 1970/IV, 1971/I and 1971/III: .050, .047 and .051.

**Marginal significance levels for 1973/IV and 1975/III: .018 and .0048.

*Table 3*

*Constrained St. Louis Equation with Dummies*

$$\dot{Y}_t = \alpha + \sum_{i=0}^{4} m_i \dot{M}_{t-i} + \sum_{i=0}^{4} e_i \dot{E}_{t-i} + \sum_{s=1970/I}^{1976/IV} \gamma_s D_{ts}$$

Almon fourth degree polynomials on $m_i$ and $e_i$ with zero endpoints

Sample period: 1953/I-1976/IV

| | | | | | |
|---|---|---|---|---|---|
| $m_0$ | .303 | ( 2.057) | $e_0$ | .0726 | ( 1.767) |
| $m_1$ | .468 | ( 5.897) | $e_1$ | .0919 | ( 3.682) |
| $m_2$ | .380 | ( 3.013) | $e_2$ | .0254 | ( .752) |
| $m_3$ | .0940 | ( 1.194) | $e_3$ | -.0914 | (-3.677) |
| $m_4$ | -.164 | (-1.100) | $e_4$ | -.157 | (-4.067) |
| $\Sigma m_i$ | 1.080 | ( 4.948) | $\Sigma e_i$ | -.0591 | (- .884) |
| $\alpha$ | 3.216 | ( 4.036) | | | |
| $\gamma_s$ | | | $\gamma_s$ | | |
| 1970/I | -1.389 | (- .410) | 1973/III | 1.095 | ( .310) |
| 1970/II | -2.853 | (- .832) | 1973/IV | 8.599 | ( 2.306)** |
| 1970/III | -2.698 | (- .809) | 1974/I | -6.459 | (-1.888) |
| 1970/IV | -7.168 | (-2.146)* | 1974/II | -2.316 | (- .681) |
| 1971/I | 8.198 | ( 2.462)* | 1974/III | -1.856 | (- .551) |
| 1971/II | - .310 | (- .086) | 1974/IV | -1.838 | (- .549) |
| 1971/III | -7.983 | (-2.305)* | 1975/I | -5.532 | (-1.634) |
| 1971/IV | -2.932 | (- .846) | 1975/II | 6.370 | ( 1.848) |
| 1972/I | 5.076 | ( 1.474) | 1975/III | 11.301 | ( 3.271)** |
| 1972/II | 1.248 | ( .340) | 1975/IV | 1.457 | ( .430) |
| 1972/III | -3.078 | (- .859) | 1976/I | 8.041 | ( 2.316)** |
| 1972/IV | .266 | ( .074) | 1976/II | 6.264 | ( 1.756) |
| 1973/I | 3.777 | ( 1.086) | 1976/III | 1.769 | ( .523) |
| 1973/II | -4.114 | (-1.159) | 1976/IV | -3.300 | (- .975) |

SS = 643.815, $R^2$ = .530, S.E. = 3.249, D.W. = 1.866, D.F. = 61.

Analysis-of-covariance Chow Test:

$F_{7,82}$ = 3.878 (p-value = .00108)

Predictive Chow test:

$F_{28,61}$ = 2.063 (p-value = .00939)

* Marginal significance levels (p-values) for 1970/IV, 1971/I and 1971/III: .036, .017 and .025.

** Marginal significance levels for 1973/IV, 1975/III and 1976/I: .025, .0018, .024.

FOOTNOTES

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1 Econometrics textbooks typically study inference only for the full-rank linear model: see, for example, Johnston (1972, Ch.5), Maddala (1977, Ch.8) or Theil (1971, Ch.3). Thus the proof obtained here may be especially convenient in teaching situations. For a proof which uses results for the non-full-rank linear model, see Dufour (1981).

2 We may note here that variations of model (4) [or (8)] have been used previously in the literature on missing observations [e.g.Bartlett (1937), Wilkinson (1960)] and in the literature on outlier detection [e.g. Gentleman and Wilk (1975), John and Draper, (1978), Belsley, kuh and Welsch (1980, Section 2.1), Draper and John (1981)]. Salkever (1976) discussed the use of dummy variables to compute predictions and prediction error variances in standard linear models, while Fuller (1980) considered this use in more complex situations. Izan (1978), following a suggestion by A. Zellner, also discussed a variant of this technique as an alternative to the «cumulative residual» methodology which is frequently employed in the finance literature (though both approaches are basically equivalent) and used it to study the announcement effects of audited and unaudited financial information. However, none of the above authors has discussed the relationship of this technique with the Chow test, which is the main purpose of this paper, nor its general applicability as an *exploratory* device for analyzing the timing of structural change. Further, the proof we give of the equivalence between dummy variable coefficients and prediction errors, as well as between the corresponding variances, is much simpler than Salkever's or Izan's proof; in particular, it does not require consideration of the inversion of a partitioned matrix.

3 For the constrained version of the model, the same test statistics are similarly significant, in fact much more strongly (see Table 3). These results are, of course, in contrast with those of Carlson (1978, p.17) who report not to have found evidence of instability after applying the Brown, Durbin and Evans (1975) techniques (though details of this analysis are not supplied). We may note here also that Seaks and Allen (1980, p.820) have reported a significant analysis-of-covariance test for the constrained equation; however, these authors considered a different sample period (1953/I-1977/IV) and did not analyze the stability of the unconstrained equation; furthermore, predictive tests are not supplied.

4 Very similar observations can be made if one considers the Almon constrained version of the model, though again the instability appears stronger in this case (see Table 3).

5 When the second subperiod is relatively large, it is possible that the total number of coefficients in equation (8) exceeds the capacity of a standard computer package. In such cases, the following procedure may be followed: subdivide the second sample in two or more subsamples; consider in turn each of these subsamples as the second subperiod in equation (8) (i.e. a dummy variable is included in the regression for each observation in this subsample), while keeping the first subperiod unchanged; estimate equation (8) for each of these modified samples (in each case, the observations in the other subsamples of the second subperiod are simply dropped from the observation matrix). The

number and sizes of these subsamples are chosen precisely small enough, that each of these regressions can be estimated using the available computer routines. It is very easy to show that this must yield exactly the same $\gamma$ coefficient estimates and the same t-statistics as running the full regression with all the dummy variables corresponding to the second subperiod.

[6] For the case where $\mathrm{rank}(X_i) = k < T_i$, $i=1,2$, such Bayesian posterior odds were given by Zellner and Siow (1979).

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