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# EXACT NONPARAMETRIC ORTHOGONALITY AND RANDOM WALK TESTS

Bryan Campbell and Jean-Marie Dufour*

*Abstract*—The hypothesis that a variable is independent of past information, such as its own past and past realizations of other observable variables, is a frequent implication of economic theory. Yet standard regression-based tests of orthogonality may not have the correct level if there is feedback from innovations to future values of the regressors. In this paper we develop nonparametric tests of orthogonality based on signs and signed ranks which are proved to reject at their nominal levels over a wide class of models admitting feedback. The tests are robust to problems of non-normality and heteroskedasticity. Further, in simulation studies of two specifications of feedback—a rational expectations model considered by Mankiw and Shapiro, and the random walk model—we find that the nonparametric tests display remarkable power. The paper concludes with an application which assesses the efficiency of survey data on interest rate expectations previously studied by Friedman.

## I. Introduction

THE hypothesis that markets are efficient implies the testable proposition that forecast errors made by the market are independent of any information available to the market when the forecast was made. This orthogonality property of efficient markets or, more generally, of rational expectations is an instance of the wider statistical issue of determining whether two time series are stochastically independent given that they are independent of other past values of the same variables. Yet standard regression testing procedures which attempt to evaluate conditional independence may reject too often, even with fairly large samples.

We have two apparently dissimilar examples in mind. The first is a simple linear regression with predetermined variables considered by Mankiw and Shapiro (1986, referred to as MS in what follows) who found by Monte Carlo techniques that the true level of the $t$-test may be considerably larger than its nominal level even for fairly large samples. Even though asymptotic inference based on a normal distribution for the $t$-statistic is correct in their specification, the finite-sample distribution of the $t$-statistic differs considerably from its asymptotic distribution. The issue has been treated further by Banerjee and Dolado (1987, 1988), Galbraith, Dolado and Banerjee (1987), Banerjee, Dolado and Galbraith (1990), as well as by Mankiw and Shapiro (1985). The second example is the random walk model without drift which has necessitated even more radical readjustment, since the $t$-statistic associated with the OLS estimate does not have the usual asymptotic normal distribution. These two examples are illustrative of a class of models which involve feedback: future values of the regressors are affected by disturbances which are contemporaneously uncorrelated with the regressors.

In this paper, we introduce nonparametric analogues of the $t$-test, based on sign and signed rank statistics, that are applicable to a specific class of feedback models including both the MS model and the random walk without drift. The sign tests are provably exact for this class of models, irrespective of the nature of feedback, even if the disturbances are asymmetric or non-normal or heteroskedastic; under the additional

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assumption of symmetry, similar results are obtained for a class of linear signed rank statistics (e.g., Wilcoxon-type statistics). Modifications of these results are obtained as well as for cases involving discrete random variables, possibly with a mass at zero. Most importantly, simulations indicate that the nonparametric tests considered have good power relative to the $t$-test, using either the asymptotic or size-corrected critical values for the MS model or the Dickey-Fuller critical values as can be found in Fuller (1976) for the random walk model. The results of this paper involve a considerable generalization of those in Campbell and Dufour (1991), where various nonparametric statistics are introduced to deal with a variant of the MS model. In particular, the nature of the allowed feedback is considerably more general and exact distributional results are established for a class of Wilcoxon-type statistics.

The paper is organized as follows. In section II we introduce the relevant test statistics in the general feedback context and derive distributional results for various sign and signed-rank statistics. In section III two specific cases illustrating such feedback are introduced, and we present Monte Carlo results on the level and power of the proposed tests applied to these two cases. A relevant application is presented in section IV: the orthogonality of forecast errors are tested using the same survey data considered by Friedman (1980). Section V offers some concluding remarks.

## II. Nonparametric Statistics in the Feedback Context

In many tests of orthogonality between two random variables, the null hypothesis asserts that a variable $Y_t$ is independent of its own past as well as past realizations of a second variate $X_t$. Our goal is to introduce tests of this assertion which are exact under very weak assumptions concerning the distribution of $Y_t$ and the relationship between $Y_t$ and $X_t$. For one group of tests, we simply assume that $Y_t$ has median zero; for the other, we suppose that the distribution of $Y_t$ is symmetric about zero. No additional assumption other than the independence of $Y_t$ with respect to the past (denoted in what follows by $I_{t-1}$) governs the relationship between $Y_t$ and $X_t$. In more precise language, we work within the framework of the following general specification involving the random variables $Y_1, \ldots, Y_n$, $X_0, \ldots, X_{n-1}$, and the corresponding information vectors $I_t = (X_0, X_1, \ldots, X_t, Y_1, \ldots, Y_t)'$, where $t = 0, \ldots, n-1$, with the convention $I_0 = (X_0)$:

$$Y_t \text{ is independent of } I_{t-1}, \quad \text{for each } t = 1, \ldots, n; \qquad (1)$$

$$P[Y_t > 0] = P[Y_t < 0], \quad \text{for } t = 1, \ldots, n. \qquad (2)$$

Assumption (1) states that $Y_t$ is independent of the past values of $Y_t$ and $X_t$, while Assumption (2) means that $Y_1, \ldots, Y_n$ have median zero. These assumptions leave open the possibility of feedback from $Y_t$ to current and future values of the $X$-variable without specifying the form of feedback. The variables $Y_t$ and $X_t$ may have discrete distributions (which includes the possibility of non-zero probability mass at zero); as well, the variables $Y_t$ need not be normal nor identically distributed. In what follows, we shall also consider the additional assumption that $Y_1, \ldots, Y_n$ have distributions symmetric about zero:

$$Y_1, \ldots, Y_n \text{ have continuous distributions symmetric about zero.} \qquad (3)$$

Clearly, the latter assumption implies (2), but the converse is not true.

In order to motivate the nonparametric statistics introduced in this paper, it is useful to consider the following linear model:

$$Y_t = \beta X_{t-1} + e_t, \qquad t = 1, \ldots, n \qquad (4)$$

where $e_t$ has the same properties as $Y_t$ in (1) and (2). Suppose we wish to test the null hypothesis that $\beta = 0$. It seems reasonable to focus on nonparametric analogues of Student's $t$-statistics, which in this environment are derived from

$$T = \frac{\hat{\beta}}{\hat{\sigma}\left(\sum_{t=1}^{n} X_{t-1}^2\right)^{-1/2}} = \frac{\sum_{t=1}^{n} Y_t X_{t-1}}{\hat{\sigma}\left(\sum_{t=1}^{n} X_{t-1}^2\right)^{1/2}} = \sum_{t=1}^{n} V_t, \qquad (5)$$

where

$$\hat{\beta} = \sum_{t=1}^{n} Y_t X_{t-1} \Big/ \sum_{t=1}^{n} X_{t-1}^2,$$

$$\hat{\sigma}^2 = \sum_{t=1}^{n} \left(Y_t - \hat{\beta} X_{t-1}\right)^2 / (n-1)$$

and

$$V_t = Y_t X_{t-1}/\hat{\sigma}\left(\sum_{\tau=1}^{n} X_{\tau-1}^2\right)^{1/2}.$$

Nonparametric procedures abstract from the specific values of $V_t$ to consider simply its sign and possibly the rank of its absolute value among $|V_1|, \ldots, |V_n|$. In such a context the denominator $\hat{\sigma}(\sum_{t=1}^{n} X_{t-1}^2)^{1/2}$ plays no role and we are led to consider the simple products $Z_t = Y_t X_{t-1}$ as the basic building block in the definition of various nonparametric statistics. More generally, to test $\beta = \beta_0$ in the context of (4), we would start with $Z_t = (Y_t - \beta_0 X_{t-1})X_{t-1}$ as the basic product. In particular, if $X_{t-1}$ is identified with $Y_{t-1}$ in (4), we can develop in this way tests of the random walk hypothesis without drift ($\beta = 1$).

A natural nonparametric analogue of the statistic $T$ is thus the sign statistic given by

$$S_0 = \sum_{t=1}^{n} u(Y_t X_{t-1}), \tag{6}$$

where $u(z) = 1$, if $z \geq 0$, and $u(z) = 0$ for $z < 0$. In this paper, we shall in fact study a more general sign statistic of the form

$$S_g = \sum_{t=1}^{n} u(Y_t g_{t-1}), \tag{7}$$

where $g_t = g_t(I_t)$, $t = 0, \ldots, n-1$, is a sequence of measurable functions of the information vector $I_t$. Clearly $S_0$ is a special case of $S_g$ obtained by taking $g_t = X_t$. The functions $g_t(\cdot)$ allow one to consider various (possibly nonlinear) transformations of the data, provided $g_t$ depends only on past and current values of $X_\tau$ and $Y_\tau$ ($\tau \leq t$). The role of such transformations will be elaborated further below.

The statistic $S_g$ is an integer-valued random variable assuming values between 0 and $n$. Under the quite general conditions described by (1) and (2), the following proposition establishes the exact distribution of $S_g$ when $Y_t$ and $g_t$ have no probability mass at zero. This result represents a considerable generalization of the main theorem of Campbell and Dufour (1991). The proofs of all the propositions given in this section can be found in the appendix. We denote by $Bi(n, p)$ the binomial distribution with number of trials $n$ and probability of success $p$.

Proposition 1: *Let $Y = (Y_1, \ldots, Y_n)'$ and $X = (X_0, \ldots, X_{n-1})'$ be two $n \times 1$ random vectors which satisfy assumptions* (1) *and* (2). *Suppose further that $P[Y_t = 0] = 0$, for $t = 1, \ldots, n$, and let $g_t = g_t(I_t)$, $t = 0, \ldots, n-1$, be a sequence of measurable functions of $I_t$ such that $P[g_t = 0] = 0$ for $t = 0, \ldots, n-1$. Then the statistic $S_g$ defined by* (7) *follows a $Bi(n, 0.5)$ distribution, i.e. $P[S_g = x] = C_n^x(1/2)^n$ for $x = 0, 1, \ldots, n$, where $C_n^x = n!/[x!(n-x)!]$.*

This distributional result obviously also holds for $S_0$. It must be stressed that $S_0$ and, more generally, $S_g$ have well-known distribution under very general conditions. The assumption $P[Y_t = 0] = P[g_t = 0] = 0$ simply means that the variables $Y_t$ and $g_t$ have no mass at zero, which of course will hold when they have continuous distributions. Otherwise, the nature of the distribution of each $Y_t$ is left open; there are no assumptions concerning the existence of moments; heteroskedasticity of unknown form is permitted; the nature of the feedback mechanism between $Y_t$ and current and future values of $X_{t+s}$ ($s \geq 0$) is not specified. As long as $Y_t$ has median 0 and is independent of the past, the sign statistics $S_0$ and $S_g$ are all binomial with mean $n/2$ and variance $n/4$. When $Y_t$ has a mean and follows a symmetric distribution, the mean and median of $Y_t$ are identical, and so the assumptions of a zero median is equivalent to that of a zero mean. For asymmetric distributions, however, this equivalence does not hold.

Under the further assumption that each $Y_t$ has a continuous distribution symmetric around zero, i.e., under (3), it is natural to introduce ranks as well. In this paper, we consider two basic types of signed rank statistics:

$$W_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) R_{1t}^{+}, \tag{8}$$

$$SR_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) R_{2t}^{+}, \tag{9}$$

where $R_{1t}^{+}$ in $W_g$ is the rank of $|Z_{1t}| \equiv |Y_t g_{t-1}|$, i.e. $R_{1t}^{+} = \sum_{j=1}^{n} u(|Z_{1t}| - |Z_{1j}|)$ the rank of $|Z_{1t}|$ when $|Z_{11}|, \ldots, |Z_{1n}|$ are put in ascending order, while $R_{2t}^{+}$ in $SR_g$ denotes the rank of $|Y_t|$ among $|Y_1|, \ldots, |Y_n|$. We also call $W_0$ and $SR_0$ the statistics obtained by taking $g_t = X_t$ in (8) and (9):

$$W_0 = \sum_{t=1}^{n} u(Y_t X_{t-1}) R_{1t}^{+}, \tag{10}$$

$$SR_0 = \sum_{t=1}^{n} u(Y_t X_{t-1}) R_{2t}^{+}. \tag{11}$$

The statistics $W_0$ and $W_g$ defined above are standard signed rank analogues of the statistics $S_0$ and $S_g$: the statistics are computed by weighting the sign of each positive product $Y_t X_{t-1}$ (or $Y_t g_{t-1}$) by the rank of its absolute value. The possibility of feedback makes it impossible to establish in general that $W_0$ and $W_g$ are distributed as a Wilcoxon signed rank variate, i.e., as $W = \Sigma_{t=1}^{n} tB_t$ where $B_1, \ldots, B_n$ are independent random variables such that $P[B_t = 0] = P[B_t = 1] = 0.5$ for $t = 1, \ldots, n$ (independent uniform Bernoulli variables on $\{0, 1\}$). A counter-example can be found in Campbell (1990). However, simulation studies indicate that $W_0$ and $W_g$ reject at their nominal levels for the two specifications of (4) considered in this paper and, consequently, these statistics are included in the empirical studies of power in the next section. Without feedback, it is easy to establish the following proposition, which slightly extends a standard result of the theory of linear signed rank tests.

Proposition 2: *Let $Y = (Y_1, \ldots, Y_n)'$ and $X = (X_0, \ldots, X_{n-1})'$ be independent $n \times 1$ random vectors such that* (1) *and* (3) *hold. Let $g_t = g_t(X)$, $t = 0, \ldots, n-1$, be a sequence of measurable functions of the vector X such that $P[g_t = 0] = 0$. Then the statistic $W_g$ defined in* (8) *is distributed like $W = \Sigma_{t=1}^{n} tB_t$, where $B_1, \ldots, B_n$ are independent uniform Bernoulli variables on* $\{0, 1\}$.

Note that $g_t$ in Proposition 2 can be a function of all the variables $X_0, \ldots, X_{n-1}$, but does not depend on $Y$. When $g_t = X_t$, the result applies to $S_0$. By contrast, exact distributional results can be established for $SR_0$ and $SR_g$ without the additional assumption that the vectors $Y$ and $X$ are independent. In the definitions of these Wilcoxon-type statistics, the absolute ranks are defined with respect to $Y_1, \ldots, Y_n$, which are mutually independent according to (1). It is this feature which is crucial in establishing the following proposition.

Proposition 3: *Let $Y = (Y_1, \ldots, Y_n)'$ and $X = (X_0, \ldots, X_{n-1})'$ be two $n \times 1$ random vectors such that* (1) *and* (3) *hold. Let also $g_t = g_t(I_t)$, $t = 0, \ldots, n-1$, be a sequence of measurable functions of $I_t = (X_0, \ldots, X_t, Y_1, \ldots, Y_t)'$ such that $P[g_t = 0] = 0$ for $t = 0, \ldots, n-1$, let $|Y| = (|Y_1|, \ldots, |Y_n|)'$, and define the sign variables $s_t = u(Y_t g_{t-1})$ for $t = 1, \ldots, n$. Then the following two properties hold:*

(a) *the signs $s_1, \ldots, s_n$ are mutually independent and, provided $|Y_t| \neq 0$ for $t = 1, \ldots, n$,*

$$P[s_t = 0 \mid |Y|] = P[s_t = 1 \mid |Y|] = 0.5, \quad \text{for } t = 1, \ldots, n;$$

(b) *the statistic $SR_g$ defined by* (9) *follows the same distribution as the Wilcoxon signed rank variate $W = \Sigma_{t=1}^{n} tB_t$, where $B_1, \ldots, B_n$ are independent uniform Bernoulli variables on* $\{0, 1\}$.

Again it is clear that the result of Proposition 3 also holds for $SR_0$ by taking $g_t = X_t$. For a general discussion of the variable $W$, see Lehmann (1975). The distribution of $W$ has been extensively tabulated (see, for example, Wilcoxon, Katti and Wilcox (1970)) and the normal approximation with $E(W) = n(n+1)/4$ and $\text{Var}(W) = n(n+1)(2n+1)/24$ works well even for small values of $n$. Proposition 3(a) also provides the basic property for establishing the distributions of more general linear signed rank statistics analogous to $SR_g$, i.e., statistics of the form

$$\sum_{t=1}^{n} u(Y_t g_{t-1}) a_n(R_{2t}^{+})$$

where $a_n(\cdot)$ is a "score" function. The distributions of such statistics, however, are not well tabulated and studying the choice of the score function is beyond the scope of the present paper. For further discussion of linear signed rank statistics, see Hájek and Šidák (1967), Dufour (1981), and Dufour and Hallin (1992, 1993).

Up to this point we have assumed that $Y_t$ and $X_t$ (or more generally $g_t$) had no probability mass at zero. In the following proposition, we relax totally or partially these assumptions.

Proposition 4: *Let $Y = (Y_1, \ldots, Y_n)'$ and $X = (X_0, \ldots, X_{n-1})'$ be two $n \times 1$ random vectors such* *that* (1) *and* (2) *hold, let* $g_t = g_t(I_t), t = 0, \ldots, n-1$, *be a sequence of measurable functions of* $I_t = (X_0, \ldots, X_t, Y_1, \ldots, Y_t)'$, *and set* $\bar{g}_t = g_t + \delta(g_t)$, *where* $\delta(x) = 1$ *if* $x = 0$, *and* $\delta(x) = 0$ *if* $x \neq 0$. *Let also* $S_g$ *and* $SR_g$ *be defined as in* (7) *and* (9), *set*

$$\bar{S}_g = \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}), \overline{SR}_g = \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}) R_{2t}^{+},$$

$\delta(Y) = [\delta(Y_1), \ldots, \delta(Y_n)]'$, *and let* $n^* = n - \sum_{t=1}^{n} \delta(Y_t)$, *the number of non-zero* $Y_t$*'s. Then the following properties hold*:

(a) $0 \leq S_g \leq \bar{S}_g, 0 \leq SR_g \leq \overline{SR}_g$, *and the conditional distribution of* $\bar{S}_g$ *given* $\delta(Y)$ *is* $Bi(n^*, 0.5)$;

(b) *if* $P[g_t = 0] = 0$ *for* $t = 0, \ldots, n-1$, *we have* $S_g = \bar{S}_g$ *and* $SR_g = \overline{SR}_g$ *with probability* 1, *and the conditional distribution of* $\bar{S}_g$ *given* $\delta(Y)$ *is* $Bi(n^*, 0.5)$;

(c) *If assumption* (3) *holds,* $\overline{SR}_g$ *follows the same distribution as the Wilcoxon signed rank variate* $W = \sum_{t=1}^{n} t B_t$, *where* $B_1, \ldots, B_n$ *are independent uniform Bernoulli variables on* $\{0, 1\}$.

Part (b) of Proposition 4 shows that, provided $g_0, \ldots, g_{n-1}$ have no probability mass at zero, tests based on $S_g$ can be performed conditionally on the non-zero $Y_t$'s, i.e. after dropping the zero $Y_t g_{t-1}$ products. For the more general case where $g_0, \ldots, g_{n-1}$ may have a mass at zero, the distribution of $S_g$ appears difficult to determine. Proposition 4(a), however, shows that a simple alternative consists in replacing $S_g$ by the closely related statistic $\bar{S}_g$, to which the result of part (b) applies. When $P[g_t = 0] = 0$ for $t = 0, \ldots, n-1$, the two statistics coincide with probability 1. Similarly under assumption (3), we can use the statistic $\overline{SR}_g$ instead of $SR_g$; by Proposition 4(c), $\overline{SR}_g$ follows the usual Wilcoxon distribution. We do not study here the distribution of $SR_g$ when $Y_1, \ldots, Y_n$ have masses at zero, because in such a situation it is a more complicated linear signed rank statistic. For a further discussion of such statistics, see again Dufour and Hallin (1992, 1993).

Given that the exact distribution of both a sign statistic and a class of signed rank statistics is known under quite general conditions, the issue of power becomes crucial in determining the usefulness of these nonparametric tests. For example, in model (4) we have

$$Y_t X_{t-1} = \beta X_{t-1}^2 + e_t X_{t-1}.$$

The sign and Wilcoxon tests based on $Z_t \equiv Y_t X_{t-1}$ test in effect whether the random variable $Z_t$ is centered at zero, more precisely whether $Z_t$ has median zero. Under the null hypothesis ($\beta = 0$), the median is determined by the behavior of $e_t X_{t-1}$ which is zero under assumption (2). When $\beta \neq 0$, the median of $Z_t$ is displaced from zero by the expression $\beta X_{t-1}^2$ and, as $\beta$ gets larger, it is expected that the displacement is more severe and the test more powerful. This intuition is confirmed by the empirical studies in the following section.

There remain two points which are relevant to the application of these nonparametric tests. First, the assumption in (1) that the disturbances are mutually independent cannot be relaxed without compromising the distributional results established in this section. But the approach can be modified to deal with certain patterns of dependence such as MA($q$) disturbances. For example, if $Y_t$ represented a two-period forecast error, it is entirely consistent with the efficiency hypothesis associated with rational forecasting that $Y_t$ behave as an MA(1) process. In this instance, the independence required to use the nonparametric procedures can be recaptured by splitting the sample into two with alternate points assigned to different subsamples. At least two simple testing strategies are then available. In the first, a nonparametric test with level $\alpha/2$ is applied to each subsample and the null hypothesis is rejected if one of the tests is significant; by Bonferroni inequality, this yields a test whose level does not exceed $\alpha$. In the second strategy, a single test with level $\alpha$ is applied to a randomly chosen subsample; this procedure is not conservative but involves dropping half the sample. Both procedures can be adapted to deal with MA($q$) disturbances or, more generally, to situations where $Y_t$ is $q$-dependent. The relative performance of these procedures with respect to power and the possibility of finding better ones are subjects of ongoing research.

Second, in many applications, it is more appropriate to consider the following variant of model (4):

$$Y_t = \beta(X_{t-1} - \mu_{t-1}) + e_t, \qquad (4')$$

where $\mu_t$ is a centering parameter for $X_t$, such as the mean, the median or the trend of $X_t$; for example, if $X_t$ is stationary, $\mu_t = \mu$ may represent the mean of $X_t$. If $Y_t$ represent forecast errors (under the null when $\beta = 0.0$, $Y_t = e_t$) which are centered at zero and $X_t$ is a macroeconomic variable which assumes only positive values, it is clear that the nonparametric statistics $S_0$, $W_0$ and $SR_0$ introduced above will have no power whatever the value of $\beta$. In this context, the rejection of the null is associated with comovements of $Y_t$ around zero and $X_{t-1}$ around its mean. However, as the proofs of Propositions 1, 2 and 4 reveal, $\mu_t$ should be estimated using only information available at time $t$ if the exact distribution of the nonparametric statistic is to be preserved. The functions $g_t(I_t)$ then represent any such estimation attempt based on partial information; various ways of centering the $X_t$ variable are considered in the application presented in section IV. For given functions $g_t(I_t)$, the sign statistic is defined in (7) and the signed rank statistics $W_g$ and $SR_g$ are given in equations (8) and (9).

## III. A Simulation Study of Two Examples

Two specifications of model (4) are now introduced to contrast the behavior of nonparametric statistics with standard regression procedures. In Mankiw and Shapiro (1986), $X_t$ is assumed to follow a stationary autoregressive process given by

$$X_t = \theta_0 + \theta_1 X_{t-1} + \epsilon_t, \qquad t = 1, \ldots, n, \tag{12}$$

where the $\epsilon_t$ are assumed to be mutually independent and each $\epsilon_t$ is independent of $X_{t-j}$, $j \geq 1$; the disturbances $e_t$ and $\epsilon_t$ are also assumed to follow a bivariate normal distribution with correlation coefficient $\rho$. It follows that $e_t$ in (4) is related to $X_t$ through $\epsilon_t$ and hence to future $X_{t+j}$ $(j > 1)$ by the autoregressive process. Since the disturbance vector $(e_1, \ldots, e_n)'$ is not independent of the explanatory variable vector $(X_0, \ldots, X_{n-1})'$, the $t$-test associated with the least squares estimate of $\beta$ in model (4) can only be justified in large samples. Mankiw and Shapiro (1986) investigated the finite-sample properties of the usual $t$-test in a Monte Carlo study and found that it over-rejects the null hypothesis when $\rho$ and $\theta_1$ are close to one and asymptotic critical points are used. By contrast, since the exact finite-sample distribution of $S_0$ and $SR_0$ are given by Propositions 1 and 3, the reliability of the associated sign and Wilcoxon tests under the null is not an issue.

To investigate empirically the relative behavior of the nonparametric versus the asymptotic regression-based procedures in the MS specification, data were generated from model (4), with the $X$ process specified as (12), by setting $\epsilon_t = \rho e_t + w_t\sqrt{1-\rho^2}$, $\theta_0 = 0$ and $X_0 = w_0/\sqrt{1-\theta_1^2}$, where $e_t$ and $w_t$ are independent with the same distribution either $N(0,1)$, $t(3)$, Cauchy, truncated Cauchy or lognormal; the asymmetric lognormal disturbances are centered at their median. Experiments illustrating the impact of heteroskedasticity involve modifications of standard normal disturbances as described in table 3. Five tests are usually considered: the non-centered $T$-test (determined by OLS regression estimates of (12) without the intercept term), the centered $t$-test (found by including an intercept in the regression) and the three nonparametric tests based on $S_0$, $W_0$ and $SR_0$, as defined in equations (6), (10) and (11). Asymptotic 5% critical values are used in applying the parametric tests; in situations where the $t$-test over-rejects, it is also applied using size-corrected critical points which are determined empirically. Note that the "size corrections" are only valid for the particular value of $\rho$ used ($\rho = 0.8$), and so the comparisons here are biased in favor of the parametric tests, since the nonparametric tests can be correctly applied without knowing the value of $\rho$. A more accurate size correction for the $t$- and $T$-tests would require one to find the supremum of the relevant critical points over all possible values of $\rho$, and even then would remain specific to the particular feedback model considered here. Because the sign and Wilcoxon statistics have discrete distributions, it is not possible (without randomization) to obtain tests whose size is precisely 5%; here, the sizes of the sign tests are 4.33%, 6.49%, 3.52%, 4.00% for $n = 25$, 50, 100, 200, respectively; for $n = 25$, 50, the levels of the Wilcoxon tests are 4.82% and 4.94%, while the normal approximation is used for the larger sample sizes. Each experiment comprises 2000 replications.

Table 1 presents simulation results based on normal disturbances for the specification given by

$\rho = 0.8$ and $\theta_1 = 0.99$ for various sample sizes. The results confirm the MS finding that asymptotic regression-based tests are unreliable when $\rho$ and $\theta_1$ are close to one: the $t$-test rejects at over twice its nominal level for sample sizes as large as 200. It is curious that the $T$-test appears to reject at its nominal level except perhaps when the sample size is small, and it should also be noted that the test based on $W_0$ rejects at its nominal level. Further, since the $T$-test does not fit a non-existent intercept (in contrast with the $t$-test), and thus is the natural parametric test to implement under the assumptions of this specification, it is to be expected that the $T$-test will exhibit better power than the $t$-test. This presumption appears to be confirmed by the results shown here. However, the most striking message of table 1 is that not much power is lost relative to the $T$-test in applying the sign test with small sample sizes or either of the Wilcoxon procedures for any sample size. Overall, the $t$-test based on corrected critical points is a poor last. Furthermore, if we were using more correct critical values for the $t$- and $T$-statistics, which should yield a probability of rejection no greater than 0.05 irrespective of the unknown value of $\rho$, the power functions of both these tests could only deteriorate with respect to the nonparametric tests.

It is possible to give a heuristic explanation of the curious result of the $T$-test having apparently correct size in table 1. Under the null hypothesis, the $T$-statistic in (5) can be written

$$T = \frac{\sum_{t=1}^{n} X_{t-1} e_t}{\left[\hat{\sigma}^2\left(\sum_{t=1}^{n} X_{t-1}^2\right)\right]^{1/2}};$$

hence for $\rho \neq 0$ and using $\epsilon_t = \rho e_t + (1 - \rho)^{1/2} w_t$,

$$T = \frac{\rho \sum_{t=1}^{n} X_{t-1}\epsilon_t + (1-\rho^2)^{1/2} \sum_{t=1}^{n} X_{t-1} w_t'}{\left[\hat{\sigma}^2 \sum_{t=1}^{n} X_{t-1}^2\right]^{1/2}},$$

where $w_t' = [(1-\rho^2)^{1/2}\epsilon_t - w_t]/\rho$ is uncorrelated with $\epsilon_t$ (and $X_t$). The second term in the latter expression for $T$ is asymptotically distributed like a normal random variable with mean zero, while

Table 1.—Mankiw-Shapiro Model: Normal Disturbances
$\rho = 0.8, \theta_1 = 0.99$
various sample sizes

| $\beta_1$ | $t$-test Asymptotic | $t$-test Size-Corrected[b] | $T$-test | $S_0$ | $SR_0$ | $W_0$ |
|---|---|---|---|---|---|---|
| | | | $n = 25$ | | | |
| 0.00 | 19.6 | 5.0 | 7.7 | 3.8 | 5.3 | 4.6 |
| 0.04 | 11.8 | 3.0 | 25.3 | 19.3 | 26.1 | 24.6 |
| 0.07 | 9.5 | 2.2 | 43.3 | 36.0 | 43.4 | 41.5 |
| | | | $n = 50$ | | | |
| 0.00 | 18.7 | 5.0 | 6.6 | 3.9 | 4.4 | 4.9 |
| 0.02 | 12.3 | 3.1 | 16.7 | 11.9 | 18.8 | 17.2 |
| 0.07 | 11.4 | 4.1 | 57.8 | 48.1 | 57.3 | 57.1 |
| | | | $n = 100$ | | | |
| 0.00 | 14.7 | 5.0 | 5.0 | 4.2 | 4.4 | 4.7 |
| 0.02 | 7.8 | 2.7 | 25.4 | 18.1 | 26.2 | 25.5 |
| 0.04 | 12.4 | 5.6 | 52.1 | 40.9 | 51.2 | 50.9 |
| | | | $n = 200$ | | | |
| 0.00 | 11.9 | 5.0 | 5.5 | 4.3 | 4.9 | 4.7 |
| 0.01 | 7.1 | 3.2 | 16.2 | 12.1 | 17.9 | 16.6 |
| 0.02 | 11.7 | 6.1 | 39.1 | 27.1 | 37.1 | 37.0 |

[a] Entries represent percentage rejections. The statistics $S_0$, $W_0$ and $SR_0$ are defined in equations (6), (10) and (11).

[b] Empirical critical points are used in power calculations. For $\beta_1 = 0$, the rejection frequency for the size-corrected $t$-test is 5.0% by construction.

the first should have a distribution similar to the Dickey-Fuller distribution with known zero intercept (since $\theta_1 = 0.99$ is close to 1). Then we can note that the tail areas associated with the critical values $\pm 1.96$ for the Dickey-Fuller distribution are roughly 0.05 for the lower tail and 0.01 for the upper (see Fuller (1976, table 8.5.2)), and thus should yield good size properties for a two-sided test.[1] On the other hand, because of the asymmetry of the Dickey-Fuller distribution, we cannot expect one-sided versions of the $T$-test to be reliable, especially $T$-tests against $\beta < 0$. $T$-tests would also need to be size-corrected in this case; assuming $\rho$ to be known, the measured power of the one-sided $T$-test against $\beta > 0$ would increase, while that against $\beta < 0$ would decrease. More importantly, a size correction that would try to take into account the unknown value of $\rho$ can only lower the power function of the $T$-test (since the asymptotic critical values roughly correspond to the case where $\rho = 0$).

Two general types of heteroskedasticity, again restricted to $(\rho, \theta_1) = (0.8, 0.99)$, with sample size $n = 100$, are considered in table 2. In the first, the variance of the underlying normal disturbances jumps from 1 to 16; the break occurs at one of three possible points ($t = 25, 50, 75$). In the second, the variability of the disturbances grows exponentially through the sample (i.e. $e_t$ is an $N(0,1)$ variable multiplied by $\exp(t)$). Along with the four statistics considered throughout the study, we consider in this context an attempt due to MacKinnon and White (1985) to correct in a general manner for heteroskedasticity through the preliminary estimation of a heteroskedastic-consistent covariance matrix which is then used in a GLS estimation of the model coefficients. Consistent quasi-$T$ and quasi-$t$ statistics ($WM$ and $wm$, respectively) can be computed and their performance is compared here with the other statistics.

The results of table 2 are interesting indeed. Both types of heteroscedasticity compromise the reliability of the four parametric tests, including the MacKinnon-White procedures, again the $W_0$ test rejects at its nominal level in all the specifications considered. Accordingly, the power performance of the parametric tests should be assessed using the (empirically) correct critical points. It is apparent that in the context of break heteroskedasticity these corrected tests are outperformed by all the nonparametric tests. Under the extreme form of exponential heteroscedasticity, the parametric tests over-reject considerably and the corresponding size-corrected tests show no power whatsoever. By contrast, the nonparametric tests behave quite well even under this extreme specification.

Table 3 presents results for homoskedastic non-normal disturbances which again show the nonparametric tests in a favorable light. The power of these tests improves when the disturbances are fat-tailed. With Cauchy disturbances, the sign and the Wilcoxon tests both outperform the parametric tests by a wide margin. We also consider Cauchy disturbances that have been truncated to exclude 0.025 of the distribution in each tail; all moments exist for such a bounded distribution and so standard central limit theorems would apply. Here the nonparametric tests continue substantially to outperform the $T$-test. Under lognormal disturbances, the sign test performs best; notice here that the signed rank tests appear to over-reject, a result which underscores the importance of the assumption of symmetry in using the signed rank procedures.

The second specification of model (4) identifies the $X$ and $Y$ processes to obtain the autoregressive model:

$$Y_t = \theta Y_{t-1} + e_t, \qquad t = 1, \ldots, n, \tag{13}$$

where the vector $(e_1, \ldots, e_n)'$ is independent of $Y_0$. We wish to test $H_0$: $\theta = 1$ (random walk without drift) against the one-sided alternative that the process is stationary ($\theta < 1$). Here, under $H_0$, the disturbances have permanent effect, and the $t$-statistic associated with the usual regression estimate of $\theta$ does not have the usual asymptotic normal distribution. To test $\theta = 1$, it will be convenient to consider the following equivalent form of (13):

$$Y_t - Y_{t-1} = \beta Y_{t-1} + e_t, \qquad t = 1, \ldots, n, \tag{13$'$}$$

where $\beta = \theta - 1$. The null hypothesis is then equivalent to $\beta = 0$, with $\beta < 0$ under the alternative. Clearly, under the null hypothesis and provided $Y_0$ and $(e_1, \ldots, e_n)'$ have continuous distributions, the assumptions of Propositions 1 and 3 are satisfied when $Y_t$ is replaced by $Y_t - Y_{t-1}$ and $X_t$ by $Y_t$. This suggests considering the

[1] We are grateful to a referee for this argument.

TABLE 2.—MANKIW-SHAPIRO MODEL: HETEROSKEDASTIC DISTURBANCES[a]
$\rho = 0.8, \theta_1 = 0.99, n = 100$

| $\beta_1$ | $t$-test | $T$-test | $wm$-test | $WM$-test | $S_0$ | $SR_0$ | $W_0$ |
|---|---|---|---|---|---|---|---|
| | | | Break at $t = 25^b$ | | | | |
| 0.00 | 12.5 | 7.8 | 11.2 | 6.0 | 3.7 | 5.4 | 5.3 |
| 0.04 | 13.4 | 37.0 | 12.5 | 32.4 | 28.2 | 36.3 | 35.5 |
| | 6.5 | 32.7 | 6.1 | 30.7 | | | |
| 0.07 | 36.2 | 63.0 | 35.1 | 59.7 | 54.6 | 60.5 | 62.0 |
| | 25.0 | 58.9 | 24.7 | 58.3 | | | |
| | | | Break at $t = 50$ | | | | |
| 0.00 | 15.2 | 10.4 | 8.8 | 5.7 | 4.0 | 4.8 | 4.5 |
| 0.04 | 20.1 | 35.2 | 13.7 | 26.5 | 31.7 | 37.1 | 36.3 |
| | 8.1 | 28.1 | 9.3 | 24.8 | | | |
| 0.07 | 38.3 | 59.3 | 30.6 | 52.1 | 55.9 | 61.7 | 63.4 |
| | 24.6 | 51.4 | 25.6 | 49.7 | | | |
| | | | Break at $t = 75$ | | | | |
| 0.00 | 24.7 | 13.9 | 8.9 | 5.8 | 4.3 | 4.9 | 4.8 |
| 0.04 | 25.8 | 35.7 | 9.3 | 21.2 | 34.5 | 40.9 | 41.9 |
| | 7.8 | 21.7 | 6.2 | 19.8 | | | |
| 0.07 | 36.1 | 59.3 | 20.2 | 46.3 | 59.4 | 66.4 | 67.5 |
| | 18.2 | 44.0 | 15.0 | 44.4 | | | |
| | | | Exponential | | | | |
| 0.00 | 89.2 | 89.4 | 12.3 | 12.2 | 3.2 | 4.4 | 4.5 |
| 0.5 | 89.1 | 89.2 | 12.9 | 12.9 | 19.6 | 18.7 | 18.5 |
| | 6.0 | 6.1 | 5.9 | 5.6 | | | |
| 0.70 | 89.2 | 89.5 | 13.6 | 13.5 | 35.7 | 32.4 | 32.1 |
| | 6.3 | 6.4 | 5.8 | 5.7 | | | |

[a] Entries represent percentage rejections; empirical critical points are used in the power calculations for the second entry in a cell. The statistics $wm$ and $WM$ are defined in the text.
[b] In the Break model, the variance of the disturbances jumps from 1 to 16 at the indicated point; in the exponential model, the variance grows exponentially with time (i.e., $e_t$ is an $N(0, 1)$ variable multiplied by $\exp(t)$).

TABLE 3.—MANKIW-SHAPIRO MODEL: NON-NORMAL DISTURBANCES[a]
$\rho = 0.8, \theta_1 = 0.99, n = 100$

| | $t$-test | | | | | |
|---|---|---|---|---|---|---|
| $\beta_1$ | Asymptotic | Size-Corrected | $T$-test | $S_0$ | $SR_0$ | $W_0$ |
| | | $t(3)$ Distribution | | | | |
| 0.00 | 15.7 | 5.0 | 5.5 | 3.4 | 4.4 | 4.9 |
| 0.02 | 8.7 | 2.9 | 24.5 | 27.9 | 33.4 | 35.7 |
| 0.07 | 35.8 | 24.6 | 76.6 | 75.3 | 80.4 | 83.6 |
| | | Cauchy Distribution | | | | |
| 0.00 | 14.2 | 5.0 | 5.4 | 3.7 | 5.7 | 5.5 |
| 0.005 | 15.8 | 7.0 | 11.3 | 51.5 | 49.7 | 56.1 |
| 0.02 | 20.8 | 12.9 | 31.8 | 88.5 | 87.9 | 91.4 |
| | | Truncated Cauchy Distribution | | | | |
| 0.00 | 15.6 | 5.0 | 5.4 | 3.3 | 4.7 | 5.0 |
| 0.02 | 8.2 | 2.3 | 24.3 | 42.7 | 42.3 | 47.3 |
| 0.03 | 9.6 | 3.6 | 38.4 | 57.0 | 56.6 | 62.1 |
| | | Lognormal Distribution | | | | |
| 0.00 | 13.9 | 5.0 | 70.1 | 3.3 | 27.4 | 22.6 |
| 0.02 | 10.1 | 4.3 | 44.0 | 47.9 | 42.6 | 44.6 |
| 0.04 | 17.8 | 10.7 | 46.6 | 76.7 | 71.5 | 76.1 |

[a] Entries represent percentage rejections. The two-sided 0.975 critical points are used to truncate the Cauchy distribution.

following statistics for testing the random walk hypothesis:

$$S_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1})Y_{t-1}],$$

$$SR_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1})Y_{t-1}]R_{3t}^{+}, \qquad (14)$$

where $R_{3t}^{+}$ is the rank of $|Y_t - Y_{t-1}|$ among $|Y_\tau - Y_{\tau-1}|, \tau = 1, \ldots, n$. The critical regions against the one-sided alternative of stationarity have the form $S_{RW} < c_1(\alpha)$ and $SR_{RW} < c_2(\alpha)$, where the critical values are determined by the distributions given in Propositions 1 and 3, respectively. As in the MS specification, we also consider a second Wilcoxon statistic based on the ranks $R_{4t}^{+}$ associated with $|(Y_t - Y_{t-1})Y_{t-1}|$:

$$W_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1})Y_{t-1}]R_{4t}^{+}. \qquad (15)$$

On the assumption that the $e_t$ are i.i.d. normal and that $Y_0 = 0$, the appropriate parametric tests to consider in this context are based on $n(\tilde{\theta} - 1)$ (the $P$-test in tables 4, 5 and 6) and the $T$-statistic both defined using the OLS estimate of $\theta$ in (13). Since these statistics are sensitive to the value of the point of departure, it is usual practice to consider tests based on $n(\hat{\theta} - 1)$ (the $p$-test) and the $t$-statistic both defined using $\hat{\theta}$, the OLS estimate of $\theta$ in the presence of an intercept term. The correct critical points for the various parametric tests have been determined by simulation; see Fuller (1976, pp. 371, 373) for the relevant tables. It should be noted that the theoretical results of the previous section establish the robustness of $S_{RW}$ and $SR_{RW}$ to the point of departure $Y_0$.

These parametric procedures were chosen simply because they are widely used in applied work. The intention in this study is to highlight the potential of nonparametric procedures in what for some may be a surprising context. We should emphasize that the nonparametric tests considered here are appropriate in testing for the null of a random walk (that is, a unit root with no serial correlation) against a stationary alternative and are not meant to test the more general hypothesis of a unit root. It would be interesting to compare our testing procedures with Cochrane (1988) who has applied variogram procedures to investigate the presence of a random walk.

To assess the relative merits of the seven parametric and nonparametric tests of the random walk null, we follow the same pattern of Monte Carlo simulation used in the analysis of the MS specification. The results are presented in tables 4, 5, and 6. For the experiments with normal and heteroskedastic errors, $Y_0$ was assumed to be standard normal under the null and, under the alternative, to be drawn from a normal distribution with the appropriate variance determined by the alternative. For the analysis with non-normal errors, $Y_0$ is taken to be zero under both the null and the alternative; for such cases, $n + 1$ observations were generated using model (13), and the

TABLE 4.—RANDOM WALK WITHOUT DRIFT: NORMAL DISTURBANCES[a]

| $\theta$ | $P$-test | $T$-Test | $p$-test | $t$-test | $S_{RW}$ | $SR_{RW}$ | $W_{RW}$ |
|---|---|---|---|---|---|---|---|
| | | | $n = 50$ | | | | |
| 1.0 | 5.5 | 5.7 | 4.1 | 5.1 | 6.7 | 6.1 | 7.3 |
| 0.97 | 7.0 | 12.1 | 7.0 | 6.6 | 11.3 | 10.6 | 13.9 |
| 0.95 | 12.4 | 18.0 | 9.5 | 7.9 | 15.8 | 14.6 | 20.0 |
| | | | $n = 100$ | | | | |
| 1.0 | 5.2 | 5.3 | 5.0 | 5.3 | 4.7 | 6.1 | 6.9 |
| 0.97 | 15.4 | 21.3 | 10.2 | 8.2 | 14.5 | 16.9 | 23.5 |
| 0.95 | 30.6 | 36.9 | 16.7 | 12.7 | 21.0 | 28.1 | 37.5 |
| | | | $n = 250$ | | | | |
| 1.0 | 5.3 | 5.4 | 4.6 | 4.7 | 4.6 | 5.1 | 6.4 |
| 0.99 | 12.0 | 18.0 | 9.1 | 7.1 | 11.5 | 16.1 | 20.5 |
| 0.98 | 30.6 | 38.0 | 18.1 | 12.7 | 20.3 | 27.8 | 37.0 |

[a] Entries represent percentage rejections. The statistics $S_{RW}$, $SR_{RW}$ and $W_{RW}$ are defined in equations (14) and (15). The critical values for the four parametric tests can be found in tables 8.5.1 and 8.5.2 in Fuller (1976).

TABLE 5.—RANDOM WALK WITHOUT DRIFT: HETEROSKEDASTIC DISTURBANCES[a]
$n = 100$

| $\theta$ | $P$-test | $T$-test | $p$-test | $t$-test | $S_{RW}$ | $SR_{RW}$ | $W_{RW}$ |
|---|---|---|---|---|---|---|---|
| | | | Break at $t = 25$ | | | | |
| 1.0 | 8.6 | 8.1 | 4.5 | 2.3 | 4.3 | 5.2 | 6.7 |
| 0.97 | 21.0 | 21.4 | 10.7 | 6.0 | 13.8 | 14.1 | 18.0 |
| | 14.0 | 14.0 | 11.7 | 12.1 | | | |
| 0.95 | 35.3 | 35.9 | 18.0 | 10.4 | 19.3 | 23.2 | 29.6 |
| | 24.2 | 24.8 | 19.1 | 20.1 | | | |
| | | | Break at $t = 50$ | | | | |
| 1.0 | 11.4 | 10.5 | 7.1 | 4.6 | 5.5 | 5.8 | 7.7 |
| 0.97 | 26.0 | 25.6 | 14.8 | 9.4 | 13.4 | 15.1 | 18.2 |
| | 11.0 | 12.0 | 11.2 | 10.2 | | | |
| 0.95 | 40.7 | 39.7 | 22.5 | 14.1 | 19.6 | 22.9 | 28.3 |
| | 21.7 | 21.0 | 17.1 | 16.1 | | | |
| | | | Break at $t = 75$ | | | | |
| 1.0 | 13.6 | 12.4 | 10.5 | 6.6 | 4.5 | 4.8 | 6.5 |
| 0.97 | 30.1 | 28.6 | 20.9 | 13.3 | 13.6 | 15.5 | 19.5 |
| | 13.7 | 14.6 | 9.8 | 10.0 | | | |
| 0.95 | 46.1 | 42.8 | 29.5 | 19.8 | 19.4 | 23.5 | 28.6 |
| | 23.6 | 24.1 | 15.9 | 14.7 | | | |
| | | | Exponential | | | | |
| 1.0 | 54.9 | 51.2 | 53.9 | 48.4 | 4.9 | 4.8 | 5.0 |
| 0.8 | 54.3 | 50.9 | 53.5 | 48.2 | 11.1 | 10.8 | 11.0 |
| | 5.7 | 5.6 | 5.7 | 5.6 | | | |
| 0.7 | 55.7 | 52.0 | 54.9 | 49.8 | 16.4 | 15.0 | 15.2 |
| | 5.7 | 6.0 | 5.6 | 6.1 | | | |

[a] Entries represent rejections; empirical critical points are used in the power calculations for the second entry in a cell. In the Break model, the variance of the disturbances jumps from 1 to 16 at the indicated point; the variance grows exponentially with with time in the exponential model ($e_t$ is an $N(0, 1)$ variable multiplied by $\exp(t)$).

TABLE 6.—RANDOM WALK WITHOUT DRIFT: NON-NORMAL DISTURBANCES[a]
$n = 100$

| $\theta$ | $P$-test | $T$-test | $p$-test | $t$-test | $S_{RW}$ | $SR_{RW}$ | $W_{RW}$ |
|---|---|---|---|---|---|---|---|
| | | | $t(3)$ Disturbances | | | | |
| 1.0 | 4.2 | 4.1 | 4.8 | 5.7 | 4.1 | 5.2 | 5.8 |
| 0.98 | 10.3 | 10.5 | 7.8 | 5.7 | 17.4 | 17.7 | 24.6 |
| 0.97 | 15.3 | 15.9 | 9.5 | 6.5 | 24.3 | 24.5 | 36.2 |
| | | | Cauchy Disturbances | | | | |
| 1.0 | 3.2 | 3.1 | 5.7 | 7.6 | 4.8 | 5.6 | 5.9 |
| 0.99 | 4.7 | 5.0 | 5.6 | 6.4 | 69.3 | 66.2 | 74.7 |
| 0.98 | 6.7 | 7.2 | 5.7 | 4.8 | 85.0 | 84.7 | 92.2 |
| | | | Truncated Cauchy Distribution | | | | |
| 1.0 | 4.6 | 5.0 | 5.4 | 5.7 | 4.6 | 5.3 | 6.8 |
| 0.99 | 7.8 | 8.0 | 7.9 | 6.0 | 15.0 | 11.4 | 16.1 |
| 0.98 | 11.7 | 12.2 | 9.9 | 6.9 | 30.0 | 23.3 | 36.8 |
| | | | Lognormal Disturbances | | | | |
| 1.0 | 0.0 | 0.0 | 0.0 | 0.1 | 4.8 | 0.0 | 0.0 |
| 0.99 | 0.0 | 0.0 | 0.0 | 5.9 | 47.7 | 4.8 | 13.5 |
| 0.98 | 0.0 | 0.0 | 2.0 | 9.0 | 78.2 | 24.4 | 50.2 |

[a] Entries represent percentage rejections. The two-sided 97.5 points are used to truncate the Cauchy distribution.

summations in (14) and (15) run from $t = 2$ to $t = n + 1$ (because the first sign variable is always zero).

The power of the nonparametric statistics as revealed in table 4 is striking. $S_{RW}$ uniformly outperforms both the centered parametric tests which are usually applied in the context of independent homoskedastic normal disturbances. For larger samples the signed rank statistic with known distribution under the null displays more power than the sign test; there is some indication that the other Wilcoxon statistic rejects somewhat more than its nominal level. In the presence of heteroskedastic disturbances (table 5), the parametric tests perform irregularly. For example, when the variance suddenly jumps at a point in the sample, the $t$-test may either be conservative if the break point is early or somewhat liberal if the point occurs later in the sample. By contrast, $S_{RW}$ and $SR_{RW}$ are reliable and show similar powers, both superior to that of the centered parametric tests.

As in the MS specification, the nonparametric statistics display considerable power in the context of fat-tailed distributions, outperforming by a considerable margin the parametric alternatives (table 6). With asymmetric lognormal disturbances, both signed rank statistics appear highly conservative; again the sign test exhibits remarkable power.

## IV. An Application

Friedman (1980) studied interest rate expectations based on survey data published by *The Goldsmith-Nagan Bond and Money Market Letter*, a publication with a wide circulation among money market professionals. Late in the concluding month of each quarter, a selected group of its subscribers were asked to forecast the values of ten interest rates on the last business day of the two following quarters. The means of the different forecasts were subsequently published along with the names of the participants in the survey. In his study based on data from 1969 to 1977, Friedman focused on six rates of the most highly traded assets; here we consider three: U.S. Treasury Bills (3-month), Utility Bonds, and Municipal Bonds. We follow Friedman in considering one aspect of the rationality hypothesis: whether the forecasters made efficient use of readily available information concerning, to use his examples, the unemployment rate, the growth rates of the consumer price index (CPI), industrial production and M1, and the federal deficit (in levels). Our goal in this section is to illustrate the details of the nonparametric approach and to compare the results with those of the more standard regression-based approach used by Friedman. A more thorough analysis of a nonparametric methodology to assess the adequacy or rationality of federal budget forecasts can be found in Campbell and Ghysels (1995).

The nonparametric statistics considered in this section have the form:

$$S_j^c = \sum_{t=1}^{n} u\left[(r_t - r_t^e) X_{t-j}^c\right],$$

$$SR_j^c = \sum_{t=1}^{n} u\left[(r_t - r_t^e) X_{t-j}^c\right] R_{1t}^+,$$

$$W_j^c = \sum_{t=1}^{n} u\left[(r_t - r_t^e) X_{t-j}^c\right] R_{2t}^+, \tag{16}$$

for $j = 1, \ldots, 4$. It should be emphasized that the Wilcoxon-type statistics are only valid under the assumption of symmetry of forecast errors. Here $r_t^e$ is the forecast of $r_t$ determined in the previous period. Further, if $r_t - r_t^e$ has an asymmetric distribution, the sign statistic provides a valid test of independence between $r_t - r_t^e$ and $X_{t-j}$ jointly with the hypothesis of a zero median (as opposed to a zero mean, suggested by several rational expectations models).[2] $X_t^c$ denotes a centered value of $X_t$ where the centering (or detrending) is based only on information available at time $t$; see the end of section II for a discussion of the motivation for using such variables. Simulation studies involving such a centering procedure in this context are presented in Campbell and Dufour (1991). In this application, we use one of two general centering methods: (1) the distance relative to a cumulative moving average of the mean, and (2) the distance from a recursively estimated linear time trend. Specifically to obtain $X_t^c$, the unemployment rate and the growth rates of the CPI, industrial production and M1 were centered using the first method. The fifth macroeconomic

[2] Median unbiasedness is entailed by rational expectations when the mean absolute forecast error is minimized rather than the mean square forecast error.

variable considered by Friedman was the Federal Deficit (in levels) and, given its evident nonstationarity, it is more appropriate to center this variable using the second method. All the variables are taken from the Main Economic Indicators of the OECD data base for a sample beginning in 1962. Each of the nonparametric statistics is then computed for $j = 1, \ldots, 4$; $p$-values (for two-sided tests) are reported in table 7. The results of a joint nonparametric test are also reported: here the null of efficiency is rejected if the smallest $p$-value (among $j = 1, \ldots, 4$) is less than 0.0125. This procedure yields a test whose level does not exceed 0.05. Table 7 presents analogous regression-based results: we consider the $t$-statistic calculated from a one-variable regression (with constant) corresponding to the nonparametric tests introduced in (16) for each $j = 1, \ldots, 4$ (i.e., the equation $(r_t - r_t^e) = \alpha + \beta X_{t-j}^c + w_t$), as well as the results of the appropriate $F$-test in a linear regression with four lags (i.e., the $F$-test of the hypothesis $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ in the regression $(r_t - r_t^e) = \alpha + \Sigma_{j=1}^4 \beta_j X_{t-j}^c + e_t$).

TABLE 7.—GOLDSMITH-NAGAN INTEREST RATE FORECASTS
NONPARAMETRIC ORTHOGONALITY RESULTS: 1-PERIOD FORECAST ERRORS

| | Lag Joint | Treasury Bills | | | | Utility Bonds | | | | Municipal Bonds | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S_j$ | $SR_j$ | $W_j$ | $t_j$ F-test | $S_j$ | $SR_j$ | $W_j$ | $t_j$ F-test | $S_j$ | $SR_j$ | $W_j$ | $t_j$ F-test |
| Unemployment rate Mean[a] | $j = 1$ | .099 | .210 | .123 | .218 | .585 | .918 | .681 | .165 | .362 | .465 | .285 | .340 |
| | $j = 2$ | .200 | .245 | .175 | .189 | .856 | .999 | .607 | .058 | .585 | .491 | .149 | .111 |
| | $j = 3$ | .362 | .465 | .267 | .189 | .999 | .622 | .666 | .015[b] | .856 | .629 | .084 | .032[b] |
| | $j = 4$ | .585 | .821 | .459 | .375 | .856 | .579 | .869 | .028[b] | .999 | .727 | .096 | .023[b] |
| | Joint | .099 | .210 | .123 | .400 | .585 | .579 | .607 | .031[b] | .362 | .465 | .084 | .134 |
| CPI Growth Rate Mean | $j = 1$ | .362 | .213 | .258 | .385 | .585 | .999 | .636 | .369 | .362 | .943 | .758 | .211 |
| | $j = 2$ | .999 | .902 | .934 | .315 | .099 | .051 | .044[b] | .030[b] | .585 | .079 | .074 | .001[b] |
| | $j = 3$ | .999 | .766 | .902 | .829 | .099 | .031[b] | .053 | .071 | .585 | .094 | .139 | .068 |
| | $j = 4$ | .585 | .329 | .365 | .322 | .999 | .334 | .537 | .751 | .999 | .614 | .696 | .244 |
| | Joint | .362 | .213 | .258 | .129 | .099 | .031 | .044 | .108 | .362 | .079 | .074 | .031[b] |
| Industrial Production (Growth Rate) mean | $j = 1$ | .855 | .781 | .523 | .986 | .200 | .061 | .092 | .149 | .099 | .006[b] | .015[b] | .104 |
| | $j = 2$ | .999 | .829 | .711 | .298 | .362 | .067 | .025[b] | .004[b] | .043[b] | .009[b] | .002[b] | .003[b] |
| | $j = 3$ | .585 | .355 | .323 | .133 | .362 | .853 | .918 | .818 | .585 | .267 | .303 | .736 |
| | $j = 4$ | .362 | .304 | .144 | .024[b] | .043[b] | .232 | .144 | .207 | .856 | .837 | .934 | .619 |
| | Joint | .362 | .304 | .144 | .068 | .043 | .061 | .025 | .025[b] | .043 | .006[c] | .002[c] | .048[b] |
| M1 Growth Rate Mean | $j = 1$ | .855 | .992 | .967 | .339 | .585 | .789 | .696 | .699 | .099 | .399 | .113 | .501 |
| | $j = 2$ | .099 | .092 | .038[b] | .141 | .043[b] | .056 | .005[b] | .074 | .099 | .136 | .025[b] | .329 |
| | $j = 3$ | .999 | .593 | .773 | 562 | .362 | .484 | .355 | .981 | .999 | .975 | .934 | .816 |
| | $j = 4$ | .200 | .053 | .113 | .042[b] | .016[b] | .003[b] | .010[b] | .074 | .200 | .159 | .241 | .661 |
| | Joint | .099 | .053 | .038 | .107 | .016 | .003[c] | .011[c] | .224 | .099 | .136 | .025 | .843 |
| Federal Deficit Regression | $j = 1$ | .200 | .241 | .181 | .369 | .362 | .294 | .294 | .215 | .200 | .217 | .144 | .430 |
| | $j = 2$ | .099 | .249 | .175 | .098 | .099 | .036[b] | .056 | .017[b] | .016[b] | .014[b] | .013[b] | .080 |
| | $j = 3$ | .043[b] | .060 | .074 | .129 | .016[b] | .007[b] | .014[b] | .007[b] | .005[b] | .002[b] | .002[b] | .018[b] |
| | $j = 4$ | .361 | .478 | .472 | .674 | .200 | .031[b] | .048[b] | .076 | .016[b] | .002[b] | .001[b] | .056 |
| | Joint | .043 | .060 | .074 | .120 | .016 | .007[c] | .048 | .026[b] | .005[c] | .002[c] | .001[c] | .104 |

Notes: The sample covers 1969:4 to 1977:1 ($n = 30$). The statistics $S_j$, $SR_j$ and $W_j$ are given in equation (16) for $j = 1$ to 4. Exact $p$-values (two-sided tests) are calculated for the sign statistics; the normal approximation is used for $SR_j$ and $W_j$. A joint nonparametric test is significant if the smallest $p$-value (among $j = 1, \ldots, 4$) is less than or equal to 0.0125. We also report the corresponding $p$-values of the $t$-statistic ($t_j$) for the explanatory variables in a regression (with intercept) of the forecast error on the indicated lag of the centered macroeconomic variable considered, as well as the $p$-value of the standard $F$-test for the joint significance of the explanatory variables associated with the appropriate regression (with intercept).

[a] The macroeconomic variables are centered recursively according to the indicated procedure: Mean corresponds to method (1) in the text; Regression, to method (3).

[b] $p$-values less than or equal to 0.05.

[c] $p$-values less than or equal to 0.0125.

Table 7 indicates there is core agreement between the parametric and nonparametric approaches. In sixteen of the twenty single-variable cases considered, neither the nonparametric nor parametric test of the efficiency of the Treasury Bills forecasts is significant and, in the other four cases, the evidence is mixed. The joint efficiency tests are not significant for all five macroeconomic variables under both approaches. In sum, there is little evidence that the information contained in the five variables is not efficiently used in the Treasury Bills forecasts. By contrast, the interest rate forecasts for Utility Bonds do not appear to be as efficient: both the nonparametric and parametric results reject the efficiency hypothesis regarding specified lags of the Federal Deficit, as well as the joint efficiency hypothesis for this variable. However, there is interesting divergence between the two approaches applied to the Utility Bonds forecasts, which are found to be inefficient with regard to the information contained in the unemployment rate only by the parametric tests and inefficient with regard to M1 only by the nonparametric tests. Such divergence is somewhat less dramatic in the analysis of the efficiency of the Municipal Bonds forecasts which are found to be inefficient according to the parametric and nonparametric approaches with regard to information contained in both Industrial Production and the Federal Deficit. It is noteworthy that in contrast to the nonparametric results the $t$-test is significant for two of the lags of the Unemployment Rate in the single-equation efficiency tests and in one lag of the CPI. These may represent examples of spurious rejection as underscored by Mankiw and Shapiro (1986). Whatever the ultimate interpretation of these results, the important point is that the nonparametric results are more credible than the regression-based alternatives.

## V. Concluding Remarks

It is a testable implication of expectations models which imply that some observed variable is a rational forecast of another unobserved variable that the forecast error is independent of information available to the forecaster. Generally, this information is not strictly exogenous, and the issue of finite-sample bias associated with the usual regression procedure arises. The sign and signed rank procedures introduced in this paper are exact in such situations under minimal assumptions of 0-median for the sign procedures and symmetry about 0 for the signed rank procedures, and are robust to problems of heteroskedasticity and non-normality. Moreover, as revealed in two simulation studies, the power of the nonparametric tests can be considerably superior to that of the parametric $t$-tests, particularly in the presence of heteroskedasticity or non-normal disturbances.

The general feedback model considered in this paper does not contain an intercept term. In the presence of such a term, $\alpha$, the methods used in this paper could readily be modified if $\alpha$ were known. Various methods of dealing with such situations are discussed in Campbell (1990). In particular, it is possible to adapt the procedures of Dufour (1990) to obtain exact nonparametric tests in the presence of the unknown nuisance parameter $\alpha$. These results will be presented in a forthcoming paper. As well, our results depend on the assumption that there is no serial correlation among the disturbances; while two approaches to specific instances of this problem were suggested in section II, the issue of dependence is the focus of ongoing research.

## APPENDIX

*Proof of Proposition 1:* Let $s_t = u(Y_t g_{t-1})$ and consider the characteristic function of $S_g$:

$$\phi_g(\tau) = E\left[\exp(i\tau S_g)\right] = E\left[\prod_{t=1}^{n} \exp(i\tau s_t)\right],$$

where $\tau \in \mathbb{R}$ and $i = \sqrt{-1}$. Conditional on the vector $I_{n-1} = (X_0, X_1, \ldots, X_{n-1}, Y_1, \ldots, Y_{n-1})'$, the variables $s_1, \ldots, s_{n-1}, g_{n-1}$ are fixed. We can thus write

$$\phi_g(\tau) = E\left\{\sum_{t=1}^{n-1} \exp(i\tau s_t) E\left[\exp(i\tau s_n)|I_{n-1}\right]\right\}.$$

When computing $E[\exp(i\tau s_n)|I_{n-1}]$, we can assume without loss of generality that $g_{n-1} \neq 0$ (an event with probability 1). Then, from (1), (2), and the assumption that $Y_n$ has no probability mass at 0, we have $P[s_n = 0|I_{n-1}] = P[s_n = 1|I_{n-1}] = 0.5$ almost everywhere. It follows that

$$\phi_g(\tau) = (0.5)[1 + \exp(i\tau)] E\left[\prod_{t=1}^{n-1} \exp(i\tau s_t)\right].$$

Applying the same argument to $E[\prod_{t=1}^{n-j} \exp(i\tau s_t)]$ for $j = 1, \ldots, n-1$, we find

$$\phi_g(\tau) = \{(0.5)[1 + \exp(i\tau)]\}^n,$$

which is the characteristic function of the binomial distribution with number of trials $n$ and probability of success 0.5. Thus $S_g$ follows a $Bi(n, 0.5)$ distribution. □

*Proof of Proposition 2:* Let $Z_{1t} = Y_t g_{t-1}, t = 1, \ldots, n$. Without loss of generality, we can only consider the case where $g_t \neq 0$ for $t = 1, \ldots, n$ (an event with probability 1). Conditional on $|g| = (|g_0|, \ldots, |g_{n-1}|)'$, the variables $Z_{1t}, t = 1, \ldots, n$, are mutually independent with $P[Z_{1t} > 0 \mid |g|] = P[Z_{1t} < 0 \mid |g|] = 0.5$; further, the rank vector $R_1^+ = (R_{11}^+, \ldots, R_{1n}^+)'$, which is a function of $|g|$, is a fixed permutation of the integers $1, 2, \ldots, n$. Conditional on $|g|$, $W_g$ has the same distribution as the Wilcoxon variate $W = \sum_{t=1}^n tB_t$. Since the distribution does not depend on $|g|$, the result also holds unconditionally. □

*Proof of Proposition 3:* (a) Let $z = (z_1, \ldots, z_n)' \in \mathbb{R}^n$ and $\bar{S}_t = \sum_{\tau=1}^t z_\tau s_\tau$ for $t = 1, \ldots, n$. The conditional characteristic function of the random vector $s = (s_1, \ldots, S_n)'$ given $|Y|$ can be written:

$$\begin{aligned}\phi_s(z) &= E\{\exp(iz's) \mid |Y|\} = E\{\exp(i\bar{S}_n) \mid |Y|\} \\ &= E\{\exp[i(\bar{S}_{n-1} + z_n s_n)] \mid |Y|\} \\ &= E\{\exp(i\bar{S}_{n-1}) E[\exp(iz_n s_n) \mid I_{n-1}, |Y_n|] \mid |Y|\}.\end{aligned}$$

When computing $E[\exp(iz_n s_n) \mid I_{n-1}, |Y|]$, we can assume that $g_{n-1} \neq 0$ (an event with probability 1). Further, by (1), (3) and the assumption that $Y_n$ has no mass at zero,

$$\begin{aligned}&E[\exp(iz_n s_n) \mid I_{n-1}, |Y_n|] \\ &\quad = E[\exp(iz_n s_n) \mid I_{n-1}] \\ &\quad = (0.5)[1 + \exp(iz_n)],\end{aligned}$$

so that

$$\begin{aligned}\phi_s(z) &= (0.5)[1 + \exp(iz_n)] \\ &\quad \times E\{\exp(i\bar{S}_{n-1}) \mid |Y|\}.\end{aligned}$$

Applying the same argument to $E\{\exp(i\bar{S}_{n-j}) \mid |Y|\}$ for $j = 1, \ldots, n-1$, we find

$$\phi_s(z) = (0.5)^n \sum_{t=1}^n [1 + \exp(iz_t)],$$

which is the characteristic function we obtain when $s_1, \ldots, s_n$ are mutually independent with uniform Bernoulli distributions over $\{0, 1\}$. Thus $s_1, \ldots, s_n$ are mutually independent conditional on $|Y|$, with $P[s_t = 0 \mid |Y|] = P[s_t = 1 \mid |Y|] = 0.5$, for $t = 1, \ldots, n$.

(b) With probability 1, we have $|Y_t| \neq 0$ for $t = 1, \ldots, n$. Conditional on $|Y|$ such that $Y_t \neq 0$ for $t = 1, \ldots, n$, the rank vector $(R_{21}^+, \ldots, R_{2n}^+)'$ is a fixed permutation of the integers $(1, \ldots, n)'$. Hence, using part (a) of the proposition, $SR_g$ conditional on $|Y|$ follows a Wilcoxon signed rank distribution $W = \sum_{t=1}^n tB_t$. Since this distribution does not depend on $|Y|$, the result also holds unconditionally. □

*Proof of Proposition 4:* It will be convenient to prove (b) first.

(b) Since $g_t$ and $g'_t$ differ only when $g_t = 0$ (an event with probability zero by assumption), it is clear that $S_g = \bar{S}_g$ and $SR_g = \overline{SR}_g$ with probability 1. By assumption (2), we have

$$\begin{aligned}&P[Y_t > 0 \mid \delta(Y_t)] \\ &\quad = P[Y_t < 0 \mid \delta(Y_t)] = 0 \quad , \quad \text{if } \delta(Y_t) = 1 \\ &\quad = 0.5, \quad \text{otherwise},\end{aligned}$$

for $t = 1, \ldots, n$. Set $p_t = P[Y_t < 0 \mid \delta(Y_t)], t = 1, \ldots, n$. Then, by assumption (1), we have when $g_{t-1} \neq 0$ an event with probability 1):

$$p_t = P[g_{t-1}Y_t > 0 \mid I_{t-1}, \delta(Y)] = P[g_{t-1}Y_t < 0 \mid I_{t-1}, \delta(Y)]$$

for $t = 1, \ldots, n$. Consider now the characteristic function of $S_g$ conditional on $\delta(Y)$: for $z \in \mathbb{R}$

$$\begin{aligned}&\phi_g[z \mid \delta(Y)] \\ &\quad = E[\exp(izs_g) \mid \delta(Y)] \\ &\quad = E\left\{\prod_{t=1}^{n-1} \exp(izs_t) E[\exp(izs_n) \mid I_{n-1}, \delta(Y)] \mid \delta(Y)\right\} \\ &\quad = [(1 - p_n) + p_n \exp(iz)] E\left\{\prod_{t=1}^{n-1} \exp(izs_t) \mid \delta(Y)\right\},\end{aligned}$$

where $i = \sqrt{-1}$, hence

$$\begin{aligned}\phi_g[z \mid \delta(Y)] &= \prod_{t=1}^n [(1 - p_t) + p_t \exp(iz)] \\ &= \{(0.5)[1 + \exp(iz)]\}^{n^*},\end{aligned}$$

where $n^*$ is the number of non-zero $Y_t$'s. Since this is the characteristic function of a $Bi(n^*, 0.5)$ variable, we can conclude that the conditional distributional of $S_g$ given $\delta(Y)$ is $Bi(n^*, 0.5)$.

(a) Since $\bar{g}_t = g_t$ when $g_t \neq 0$ and $\bar{g}_t = 1$ when $g_t = 0$, we have $u(Y_t g_{t-1}) \leq u(Y_t \bar{g}_{t-1})$ for all $t$, hence

$$\begin{aligned}0 \leq S_g &= \sum_{t=1}^n u(Y_t g_{t-1}) \leq \sum_{t=1}^n u(Y_t \bar{g}_{t-1}) = \bar{S}_g, \\ 0 \leq SR_g &= \sum_{t=1}^n u(Y_t g_{t-1}) R_{2t}^+ \\ &\leq \sum_{t=1}^n u(Y_t \bar{g}_{t-1}) R_{2t}^+ = \overline{SR}_g.\end{aligned}$$

Further, $\bar{S}_g$ satisfies all the assumptions of part (b), so that its conditional distribution given $\delta(Y)$ is $Bi(n^*, 0.5)$.

(c) Since $g'_t \neq 0$ for all $t$, the distribution of $\overline{SR}_g$ follows from Proposition 3(b). □

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