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ECONOMETRICS 1
EXERCISES 3

Linear regression

1. State and prove the Gauss-Markov theorem.

2. Suppose we have
$$y = X\beta + \varepsilon \tag{1}$$
along with assumptions of the classical linear model with $E[\varepsilon\varepsilon'] = \sigma^2 I_T$. Instead of (1), we estimate β in the context of a regression with additional explanatory variables Z :
$$y = X\beta + Z\gamma + \varepsilon_* = X_*\delta + \varepsilon_* \tag{2}$$
where Z is a fixed $T \times k_0$ matrix, the extended matrix $X_* = [X, Z]$ has rank $k + k_0$ and
$$\delta = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}. \tag{3}$$
Let $\hat{\beta} = (X'X)^{-1}X'y$ and $\hat{\delta} = (X_*'X_*)^{-1}X_*'y = (\hat{\beta}_*', \hat{\gamma}_*')'$ where $\hat{\beta}_*$ is a $k \times 1$ vector.

   (a) Show that $\hat{\beta}$ and $\hat{\beta}_*$ are unbiased estimators of β.
   (b) Give the covariance matrices of $\hat{\beta}$ and $\hat{\beta}_*$ under the assumptions the classical linear model (1).
   (c) Show that $\hat{\beta}$ is more efficient than $\hat{\beta}_*$ in terms of its squared error.
   (d) Discuss the consequences of adding irrelevant explanatory variables on the estimation of regression coefficients.
   (e) Let $\hat{\varepsilon}_* = y - X_*\hat{\delta}$. Find the expected value of the sum of squares $\hat{\varepsilon}_*'\hat{\varepsilon}_*$.
   (f) Propose an unbiased estimator of σ^2.