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ADVANCED ECONOMETRIC THEORY
EXERCISES 13

GENERAL ASYMPTOTIC TESTS

1. Let $\hat{\theta}_n$ be an estimator obtained by maximizing an objective function $L_n(\theta)$, where $\theta \in \Theta \subseteq \mathbb{R}^p$, and $\theta_0 \in \Theta$ is a value of θ such that

$$n^{-1/2} \frac{\partial L_n}{\partial \theta}(\theta_0) \underset{n\to\infty}{\overset{d}{\to}} N[0, I_0]$$

and

$$-\frac{1}{n} \frac{\partial^2 L_n}{\partial \theta \, \partial \theta'}(\theta_0) \underset{n\to\infty}{\to} J_0 \text{ with probability 1}$$

where I_0 and J_0 are deterministic positive definite matrices. We wish to test the mixed hypothesis

$$H_0 : \{\theta \mid \exists\, a \in A \subseteq \mathbb{R}^q : g(\theta, a) = 0\}$$

where g is a differentiable function taking its values in $\mathbb{R}^r$, such that

$$\text{rank}\left[\frac{\partial g}{\partial \theta'}\right] = r \ , \ \text{rank}\left[\frac{\partial g}{\partial a'}\right] = q.$$

(a) Describe the following criteria for testing H_0 :

(1) the Wald statistic,

(2) the Lagrange multiplier statistic,

(3) the score statistic,

(4) the "likelihood ratio" statistic.

(b) Under usual regularity conditions, what are the asymptotic distributions of these criteria?

(c) What form takes the Wald statistic in the case of an implicit constraint (without an auxiliary parameter a)? Which important execution difference do you observe?

(d) Which important condition must be satisfied for a "likelihood ratio" statistic to follow a χ^2 distribution?

(e) What are the forms of the different statistics when $I_0 = J_0$?

(f) Provide a condition under which the Lagrange multiplier and score statistics are identical.

2. Let $h\left(Y_i, X_i, \theta_0\right), i = 1, \ldots, n$, be functions (taking their values in $\mathbb{R}^H$) such that

$$E\left[h\left(Y_i, X_i, \theta_0\right)\right] = 0,\ i = 1,\ \ldots\ ,\ n,$$

where the observations $\left(Y_i, X_i\right), i = 1, \ldots, n$ are i.i.d. If we estimate this system by the generalized method of moments, describe a specification test for this system.

Reference: Gouriéroux and Monfort (1995, Chapter 18).

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K.