

ECONOMETRIC THEORY
EXERCISES 1
MODELS

Reference: Gouriéroux and Monfort (1995, Chapter 1)

1. (a) Define the notion of *statistical model*.
(b) Explain the distinction between a *dominated statistical model* and a *homogeneous statistical model*.
(c) When is a model *nested* by another model? What is a *submodel*? What a *nesting model*?
2. (a) Explain what is an *exponential statistical model*.
(b) Give two examples of exponential statistical models and explain why these models belong to the exponential family.
(c) Is a linear model always an exponential model?
(d) Which ones of the following terms apply to exponential models: parametric, nonparametric, semiparametric?
(e) Which ones of the following terms apply to linear models: parametric, nonparametric, semiparametric?
3. Explain the difference between the Bayesian approach and the empirical Bayesian approach to the introduction of *a priori* information.
4. Let P and P^* be two probability distributions possessing densities with respect to the same measure μ .
 - (a) Define the Kullback discrepancy between P and P^* .
 - (b) Prove that:
 - i. $I(P | P^*) \geq 0$;
 - ii. $I(P | P^*) = 0 \iff P = P^*$.
5. Let $y = (y_1, \dots, y_n)'$ be a vector of observations. To explain y , we consider the linear model:

$$y = m + u, \quad m \in L, \quad u \sim N[0, \sigma^2 I_n]$$

where L is a vector \mathbb{R}^n with k . If the true probability distribution of y is $N[m_0, \sigma_0^2 I_n]$, find the pseudo true values m_0^* , σ_0^* of m and σ^2 . [I_n represents the identity matrix of order n .]

6. Consider the following simple Keynesian model:

$$\begin{aligned} C_t &= aR_t + b + u_t, \\ Y_t &= C_t + I_t, \\ R_t &= Y_t, \end{aligned}$$

where C_t represents consumption (at time t), R_t income, Y_t production, I_t investment, and u_t is a random disturbance.

- (a) Find the reduced form of this model.
- (b) Is a coherency condition needed to derive this reduced form? If yes, which one and why?
- (c) Does this model contain *latent* variables? If so, which ones?
- (a) Explain the notion of *exogeneity* with respect to a parameter.

7. Consider the following simplified equilibrium model:

$$\begin{aligned} D_t &= \alpha + 2p_t + u_{1t}, \\ S_t &= c + u_{2t}, \\ Q_t &= D_t = S_t, \quad t = 1, \dots, T \end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, and Q_t the quantity produced and sold. We suppose that the vectors $(u_{1t}, u_{2t})'$, $t = 1, \dots, T$, are independent and $N[0, I_2]$.

- (a) Find the reduced form of this model.
 - (b) For which parameters is the vector $Q = (Q_1, \dots, Q_T)'$ exogenous? Justify your answer.
 - (c) For which parameters is the vector $p = (p_1, \dots, p_T)'$ exogenous? Justify your answer.
 - (d) Are the variables Q_t and p_t simultaneous?
8. Prove the equivalence between non-causality in the sense of Granger and non-causality in the sense of Sims. (Define clearly these two notions.)
9. Give a sufficient condition under which *sequential exogeneity* is equivalent to *exogeneity* (for a parameter α) and justify your answer.

10. Consider the following equilibrium model:

$$\begin{aligned} Q_t &= a + bp_t + u_{1t}, \\ p_t &= c + dp_{t-1} + u_{2t}, \quad t = 1, \dots, T \end{aligned}$$

p_0 is fixed

where the disturbances $(u_{1t}, u_{2t})'$, $t = 1, \dots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \dots, p_T)'$

- (a) sequentially exogenous?
- (b) exogenous?
- (c) strongly exogenous?
- (d) Further, does Q_t cause p_t in the sense of Granger?

Justify your answers.

11. Consider the following equilibrium model:

$$\begin{aligned} Q_t &= a + bp_{t+1} + u_{1t}, \\ p_t &= c + dp_{t-1} + u_{2t} \quad , t = 1, \dots, T \\ p_0 &\text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t}, u_{2t})'$, $t = 1, \dots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold and p_t the price. For which parameters is the vector $p = (p_1, \dots, p_T)'$

- (a) exogenous for (a, b) ?
- (b) exogenous for (c, d) ?
- (c) sequentially exogenous for (a, b) ?
- (d) sequentially exogenous for (c, d) ?
- (e) strongly exogenous for (a, b) ?
- (f) strongly exogenous for (c, d) ?

Justify your answers.

12. Consider the following equilibrium model:

$$\begin{aligned} Q_t &= a + bp_t + u_{1t}, \\ p_t &= c + dQ_{t-1} + u_{2t}, \\ Q_0 &\text{ is fixed} \end{aligned}$$

where the disturbances $(u_{1t}, u_{2t})'$, $t = 1, \dots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \dots, p_T)'$

- (a) exogenous for (a, b) ?
- (b) exogenous for (c, d) ?

- (c) sequentially exogenous for (a, b) ?
- (d) sequentially exogenous for (c, d) ?
- (e) strongly exogenous for (a, b) ?
- (f) strongly exogenous for (c, d) ?

Justify your answers.

13. Consider the following equilibrium model:

$$\begin{aligned} D_t &= a + bp_t + u_{1t}, \\ S_t &= c + dp_{t-1} + ex_t + fx_{t-1} + u_{2t}, \\ Q_t &= D_t = S_t \quad , t = 1, \dots, T \end{aligned}$$

where D_t is the demand for a product, S_t the supply for the same product, Q_t the quantity produced, x_t is an exogenous variable, p_0 and x_0 are fixed, and $u_t = (u_{1t}, u_{2t})'$ is random vector such that $E(u_t) = 0$.

- (a) Give the structural form associated with this model.
- (b) Give the reduced form of this model.
- (c) Find the short-term multipliers for p_t and Q_t .
- (d) Find the final form of the model.
- (e) Find the dynamic multipliers for p_t .
- (f) Find the long-run form of the model and the long-term multipliers for p_t and Q_t .

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.