



## Preface

## Editorial introduction: Heavy tails and stable Paretian distributions in empirical finance<sup>☆</sup>

### A volume honoring Benoît B. Mandelbrot

This special issue is a collection of selected papers presented at the 2005 Deutsche Bundesbank Fall Conference on “Heavy tails and stable Paretian distributions in finance and macroeconomics” as well as several other related contributions from conference participants. This conference was held in celebration of the 80th birthday of Professor Benoît B. Mandelbrot and took place at the Training Centre of the Deutsche Bundesbank in Eltville, Germany, from 10 to 12 November 2005. The aims of the conference were to discuss new theoretical developments in Professor Mandelbrot’s approach and to present new applications in the fields of risk analysis, exchange rates analysis and portfolio management, and to bring these highly academic issues closer to practitioners in general—as well as to encourage the use of this approach throughout the world of central banking.

In 1963 Mandelbrot wrote<sup>1</sup> “... the empirical distributions of price changes in a speculative market are usually too *peaked* to be relative to samples from Gaussian populations. ... But there are typically so many *outliers*. ... I shall replace the Gaussian distributions throughout by another family of probability laws to be referred to as *stable Paretian*.” He therefore proposed substituting the Brownian motion of Bachelier (Bachelier, 1900), namely the stable Gaussian law, for financial data resulting from such a speculative market with the Lévy motion (Lévy, 1937), namely the stable Paretian law<sup>2</sup>. A strong argument in favor of the  $\alpha$ -stable distribution as a distributional assumption for heavy-tailed empirical data is that only the  $\alpha$ -stable distribution can serve as the limiting distribution of sums of independent identically distributed random variables proved by Zolotarev (1986). Mandelbrot (1963) also documented the most important statistical property of financial return processes, volatility clustering, as “... large changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes ...” To capture this property, Engle (1982) almost two decades later, introduced the autoregressive conditional heteroskedastic model which is—without doubt—the leading financial volatility model today and probably will be in the future, too. Three decades later, Ding et al. (1993) found that the strongest long-memory property is exhibited by absolute returns. Because of the two properties of financial return processes, extreme occurrences and volatility clustering, which are basically two sides of the same coin as analytically shown in Diebold (1988), Mandelbrot was aware of the fractal risk in the financial markets. To find empirical evidence for the fractal risks, we need not search for very long, because we are currently experiencing it—the current financial crisis, alongside Black Monday 1987, the Asian crisis in the mid 1990s and so on. Regarding the severity of the current financial crisis, the message of Mandelbrot is and will be of extreme relevance for academics and practitioners alike.

Central banks are responsible for monetary policy, which relies heavily on financial markets and institutions, and have a strong interest in the stability of the financial system. Therefore, it is important for them to understand the basic principles which drive the financial market. Mandelbrot stated in a newspaper interview<sup>3</sup> and also in his recent book<sup>4</sup> that risk assessments even under realistic assumptions cannot prevent large shocks in financial markets, but that they can enable us to prepare for such large shocks. In his conference speech A. Weber, President of the Deutsche Bundesbank, noted that “... given a possible causal relationship between monetary policy and financial markets I, as a central banker, would like to interpret the lesson to be learned from Professor Mandelbrot’s work as a warning against extreme events. ... Understanding Professor Mandelbrot’s fractal finance means being aware of the fact that we are confronted with many more bubbles, ruins and extreme outliers than the conventional theory tells us. Therefore, Professor Mandelbrot recommends that we central bankers construct monetary policy, banking supervision and market portfolio operations like Noah’s ark—a ship that is capable of withstanding the flood in the risky financial and economic sea.”

<sup>☆</sup> The views expressed in this paper are those of the authors, and not necessarily those of the Deutsche Bundesbank.

<sup>1</sup> The variation of certain speculative prices, *Journal of Business* 36, 394–419.

<sup>2</sup> Or rather  $\alpha$ -stable law with  $0 < \alpha < 2$  as a generalization of the stable Gaussian law. If  $\alpha = 2$ , it is stable Gaussian.

<sup>3</sup> *Frankfurter Allgemeine Zeitung*, 4 June 2005.

<sup>4</sup> *The (Mis)Behaviour of Markets: A Fractal View of Risk, Ruin and Reward*, Basic Books, New York, 2004, co-written with R.L. Hudson.

This special issue comprises six papers. The first two papers in the following directly concern the  $\alpha$ -stable distributions, while the others also allow for other distributions. Despite the different distributional assumptions and/or various aspects presented in the six papers, the observations made by Mandelbrot on speculative markets described above are the unifying theme of the six papers included in this issue.

The paper by Dufour and Kurz-Kim proposes a new method of estimating and testing the stability parameter. The new method enables us to use an optimal number of tail observations resulting from a theoretical or simulated relationship in practice in which the true  $\alpha$  is unknown. Therefore, it removes a severe drawback of the Hill estimator in terms of empirical relevance. Furthermore, it provides an exact confidence interval for the estimate of  $\alpha$  for finite samples. Because statistical inference in estimation and hypothesis testing under the  $\alpha$ -stable distributional assumption depend crucially on  $\alpha$ , the contribution of the paper is of importance for empirical analysis.

In the spirit of Mandelbrot, Ortobelli, Rachev and Fabozzi apply stable Paretian models to risk management, especially optimal dynamic portfolio selection. One of the main contributions of the paper is the analysis of the impact of the distributional assumptions on multi-period asset allocation decisions. The authors consider a closed-form solution to dynamic portfolio selection problems of risk-averse investors and apply it to simulated and empirical data. The main conclusion of the paper is that the stable Paretian assumption allows for a better prediction of the potential losses than distributions with thinner tails.

In practice, the stable Paretian distributions are usually observed in high-frequency data. Using some ultrahigh-frequency datasets, Chaboud, Chiquoine, Hjalmarsson and Loretan investigate empirically how frequently certain foreign exchange and U.S. Treasury security returns can be sampled without contaminating estimates of their integrated volatility with market microstructure noise. They argue that noise introduced by such effects should be less noticeable, and hence *ceteris paribus* it should be possible to sample returns more frequently for asset prices generated in very deep and liquid financial markets. Their main finding is that the standard estimator of realized volatility does not appear to suffer from market microstructure induced biases at sampling frequencies that are much higher than generally recommended in the empirical literature on estimating the realized volatility of individual stocks.

Hartmann, Straetmans and de Vries analyze the multivariate dependency structure in affine models of foreign exchange rate returns to explain the cross-sectional return dependency during crises. In line with the statistical property of the asymptotic tail dependence of heavy-tailed distributions, they study the observed strong crisis spillovers. They argue that the systemic stability of the (international) financial system hinges critically on the type of marginal distribution. Assuming the Student's *t*-distributions for the marginal distribution they specify strong crisis linkages between different currencies.

Chen, Härdle and Spokoiny propose a method under the generalized hyperbolic distributional assumption to convert the high-dimensional financial risk factors to univariate components by means of independent component analysis, thus overcoming the limitations in multivariate risk analysis prompted by the Basel Committee on Banking Supervision, the (stable) Gaussian distributional assumption and the numerical difficulty when implementing a multivariate risk analysis. They calculate some risk measures empirically using the new method in order to compare them with those based on alternative risk management models and show that the new method produces better results.

Gencay and Gradojevic consider a time-dependent entropic measure of dispersion identifying the belief heterogeneity which can potentially capture the long-range time dependency. They use this time-dependent entropy to gain insight into the evolution of the aggregate market expectations and obtain an early indication of upcoming crises or bubbles. In the empirical application, they examine the degree of belief heterogeneity of the S&P 500 market participants before the crash of 19 October 1987, and were able to identify strong abnormal shifts in the S&P 500 market participants' aggregate beliefs roughly two months prior to the October 1987 crash.

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