

User Manual for the Linux/Unix Gauss Version of BACC
(Bayesian Analysis, Computation, and Communication)

Hülya Eraslan

William McCausland

John J. Stevens

October 29, 2003

Important Information

About this Manual

This manual describes the software developed in connection with the project Bayesian Communication in the Social Sciences, Siddhartha Chib and John Geweke principle investigators. Acknowledgement in any resulting published work would be appreciated. This project was supported, in part, by Grants SBR-9600040 and SBR-9731037 from the National Science Foundation.

Help Keep This Software Free

The National Science Foundation supports this software and its continued development. It is important that we document the use of BACC. We respectfully request that all publications and working papers reporting the results of research using BACC software, include the following acknowledgement and reference:

Computations reported in this paper were undertaken [in part] using the Bayesian Analysis, Computation and Communication software (<http://www.econ.umn.edu/~bacc>) described in:

Geweke, J. (1999) "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communication" (with discussion and rejoinder), *Econometric Reviews* 18: 1-126.

BACC Software and Documentation

BACC software and documentation is available on the web at

<http://www.econ.umn.edu/~bacc>

Please send any comments or questions to

bacc@econ.umn.edu

Contents

Important Information	3
1 Getting Started with BACC	11
1.1 Introduction	11
1.2 Requirements	11
1.3 Installation and Configuration	12
2 Models	13
2.1 Introduction	13
2.1.1 Dimension parameters	13
2.1.2 Unknown Quantities	13
2.1.3 Known Quantities	13
2.1.4 Data Generating Process	14
2.1.5 Prior Distribution	14
2.1.6 Creating a Model Instance	14
2.1.7 Sampling Algorithms	14
2.1.8 Marginal Likelihood	14
2.2 The Normal Linear Model	15
2.2.1 Dimension parameters	15
2.2.2 Unknown Quantities	15
2.2.3 Known Quantities	15
2.2.4 Data Generating Process	15
2.2.5 Priors	15
2.2.6 Creating a Model Instance	16
2.2.7 Sampling Algorithms	16
2.3 The Seemingly Unrelated Regressions Model	17
2.4 The I.I.D. Finite State Model	18
2.4.1 Dimension parameters	18
2.4.2 Unknown Quantities	18
2.4.3 Known Quantities	18
2.4.4 Data Generating Process	18
2.4.5 Priors	18
2.4.6 Creating a Model Instance	18
2.4.7 Sampling Algorithms	18

2.4.8	Marginal Likelihood	19
2.5	The Non-Stationary First Order Markov Finite State Model	20
2.5.1	Dimension parameters	20
2.5.2	Unknown Quantities	20
2.5.3	Known Quantities	20
2.5.4	Data Generating Process	20
2.5.5	Priors	20
2.5.6	Creating a Model Instance	21
2.5.7	Sampling Algorithms	21
2.5.8	Marginal Likelihood	21
2.6	The Stationary First Order Markov Finite State Model	22
2.6.1	Dimension parameters	22
2.6.2	Unknown Quantities	22
2.6.3	Known Quantities	22
2.6.4	Data Generating Process	22
2.6.5	Priors	22
2.6.6	Creating a Model Instance	23
2.6.7	Sampling Algorithms	23
2.7	The Poisson Model	24
2.7.1	Dimension parameters	24
2.7.2	Unknown Quantities	24
2.7.3	Known Quantities	24
2.7.4	Data Generating Process	24
2.7.5	Priors	24
2.7.6	Creating a Model Instance	24
2.7.7	Sampling Algorithms	24
2.7.8	Marginal Likelihood	25
2.8	The Uniform Model	26
2.8.1	Dimension parameters	26
2.8.2	Unknown Quantities	26
2.8.3	Known Quantities	26
2.8.4	Data Generating Process	26
2.8.5	Priors	26
2.8.6	Creating a Model Instance	26
2.8.7	Sampling Algorithms	27
2.8.8	Marginal Likelihood	27
2.9	A Univariate Linear Model with Normal Disturbances	28
2.10	The Dichotomous Choice Model (with normally distributed disturbances)	30
2.10.1	Dimension parameters	30
2.10.2	Unknown Quantities	30
2.10.3	Known Quantities	30
2.10.4	Data Generating Process	30
2.10.5	Priors	30
2.10.6	Creating a Model Instance	30
2.10.7	Sampling Algorithms	31

2.11	The Censored Linear Model (with normally distributed disturbances)	32
2.11.1	Dimension parameters	32
2.11.2	Unknown Quantities	32
2.11.3	Known Quantities	32
2.11.4	Data Generating Process	32
2.11.5	Priors	32
2.11.6	Creating a Model Instance	32
2.11.7	Sampling Algorithms	33
2.12	The Univariate Latent Linear Model (with normally distributed disturbances)	34
2.12.1	Dimension parameters	34
2.12.2	Unknown Quantities	34
2.12.3	Known Quantities	34
2.12.4	Data Generating Process	34
2.12.5	Priors	34
2.12.6	Creating a Model Instance	34
2.12.7	Sampling Algorithms	35
2.13	A Univariate Linear Model with Student t Disturbances	36
2.14	The Dichotomous Choice Model (with Student t distributed disturbances)	38
2.14.1	Dimension parameters	38
2.14.2	Unknown Quantities	38
2.14.3	Known Quantities	38
2.14.4	Data Generating Process	38
2.14.5	Priors	38
2.14.6	Creating a Model Instance	38
2.14.7	Sampling Algorithms	39
2.15	The Censored Linear Model (with Student t distributed disturbances)	40
2.15.1	Dimension parameters	40
2.15.2	Unknown Quantities	40
2.15.3	Known Quantities	40
2.15.4	Data Generating Process	40
2.15.5	Priors	40
2.15.6	Creating a Model Instance	40
2.15.7	Sampling Algorithms	41
2.16	The Univariate Latent Linear Model (with Student t distributed disturbances)	42
2.16.1	Dimension parameters	42
2.16.2	Unknown Quantities	42
2.16.3	Known Quantities	42
2.16.4	Data Generating Process	42
2.16.5	Priors	42
2.16.6	Creating a Model Instance	42
2.16.7	Sampling Algorithms	43

2.17	A Univariate Linear Model with Finite Mixtures of Normals Disturbances . . .	44
2.18	The Dichotomous Choice Model (with a scale mixture of normals distribution for the disturbances)	46
2.18.1	Dimension parameters	46
2.18.2	Unknown Quantities	46
2.18.3	Known Quantities	46
2.18.4	Data Generating Process	46
2.18.5	Priors	46
2.18.6	Creating a Model Instance	47
2.18.7	Sampling Algorithms	47
2.19	The Censored Linear Model (with a scale mixture of normals distribution for the disturbances)	48
2.19.1	Dimension parameters	48
2.19.2	Unknown Quantities	48
2.19.3	Known Quantities	48
2.19.4	Data Generating Process	48
2.19.5	Priors	48
2.19.6	Creating a Model Instance	49
2.19.7	Sampling Algorithms	49
2.20	The Univariate Latent Linear Model (with a scale mixture of normals distribution for the disturbances)	50
2.20.1	Dimension parameters	50
2.20.2	Unknown Quantities	50
2.20.3	Known Quantities	50
2.20.4	Data Generating Process	50
2.20.5	Priors	50
2.20.6	Creating a Model Instance	51
2.20.7	Sampling Algorithms	51
2.21	An Autoregression Model	52
2.21.1	Dimension Parameters	52
2.21.2	Unknown Quantities	52
2.21.3	Known Quantities	52
2.21.4	Data Generating Process	52
2.21.5	Prior Distribution	53
2.21.6	Creating a Model Instance	53
2.21.7	Sampling Algorithm	53
2.22	An Autoregression Model with State Dependant Means	54
2.22.1	Dimension Parameters	54
2.22.2	Unknown Quantities	54
2.22.3	Known Quantities	54
2.22.4	Data Generating Process	55
2.22.5	Prior Distribution	55
2.22.6	Creating a Model Instance	56
2.22.7	Sampling Algorithm	56

3	BACC Commands	57
3.1	Overview of BACC Commands	57
3.2	Miscellaneous Gauss Issues	58
3.2.1	Running BACC within Gauss	58
3.2.2	BACC's use of Global Variables	59
3.3	Miscellaneous Gauss Issues	59
3.3.1	Running BACC within Gauss	59
3.3.2	BACC's use of Global Variables	59
3.4	Detailed Description of Commands	59
3.4.1	The <code>dirichletSim</code> Command	60
3.4.2	The <code>expect1</code> Command	61
3.4.3	The <code>expectN</code> Command	64
3.4.4	The <code>extract</code> Command	66
3.4.5	The <code>gammaSim</code> Command	67
3.4.6	The <code>gaussianSim</code> Command	68
3.4.7	The <code>listModelSpecs</code> Command	69
3.4.8	The <code>listModels</code> Command	70
3.4.9	The <code>miDelete</code> Command	71
3.4.10	The <code>miLoad</code> Command	72
3.4.11	The <code>miLoadAscii</code> Command	73
3.4.12	The <code>miSave</code> Command	74
3.4.13	The <code>miSaveAscii</code> Command	75
3.4.14	The <code>minst</code> Command	76
3.4.15	The <code>mlike</code> Command	77
3.4.16	The <code>paretoSim</code> Command	79
3.4.17	The <code>postfilter</code> Command	80
3.4.18	The <code>postsim</code> Command	81
3.4.19	The <code>postsimHM</code> Command	82
3.4.20	The <code>priorRobust</code> Command	83
3.4.21	The <code>priorfilter</code> Command	85
3.4.22	The <code>priorsim</code> Command	86
3.4.23	The <code>setseedconstant</code> Command	87
3.4.24	The <code>setseedtime</code> Command	88
3.4.25	The <code>weightedSmooth</code> Command	89
3.4.26	The <code>wishartSim</code> Command	91
4	A BACC Tutorial	93
4.1	Linking to BACC	94
4.2	Loading Known Quantities	94
4.3	Creating a Model Instance	94
4.4	Simulating the Model	94
4.5	Computing Marginal likelihoods	95
4.6	Computing Moments of Functions of Interest	96
4.7	Sensitivity to the Prior	97
4.8	Kernel Smoothing of Simulations	97
4.9	Plotting	98

4.10	Saving and Loading Model Instances	98
4.11	Drawing from Various Distributions	98
4.11.1	Dirichlet	98
4.11.2	Gamma	99
4.11.3	Gaussian (Multivariate Normal)	99
4.11.4	Pareto	99
4.11.5	Wishart (Multivariate Chi-squared)	99
A	Distributions	101
A.1	The Dirichlet Distribution	101
A.2	The Gamma Distribution	101
A.3	The Normal Distribution	102
A.4	The Pareto Distribution	102
A.5	The Poisson Distribution	102
A.6	The Wishart Distribution	103
B	A Brief Gauss Tutorial	105
B.1	Manipulating Matrices	105
B.2	Other Useful Information	107
	Bibliography	109

Chapter 1

Getting Started with BACC

1.1 Introduction

The BACC software provides the user several commands for doing Bayesian analysis and communications. This document describes the function of these commands and their inputs and outputs. It also outlines some of the theory behind the commands, and provides references to the relevant literature.

The following versions of the BACC software and documentation are available:

- Windows Matlab
- Linux/Unix Matlab
- Windows Splus
- Linux/Unix Splus
- Windows R
- Linux/Unix R
- Windows Gauss
- Linux/Unix Gauss
- Windows Console
- Linux/Unix Console

This particular manual is for the Linux/Unix Gauss version of BACC.

1.2 Requirements

The Linux/Unix Gauss version of the BACC software requires Gauss for Linux or Unix.

1.3 Installation and Configuration

1. Download the file `bacc.zip` from the BACC website,

```
http://www.econ.umn.edu/~bacc
```

and place it in your home directory or in another directory of your choosing.

2. Unzip the file `bacc.zip` by typing

```
unzip bacc.zip
```

3. Change directories by typing

```
cd Bacc/Build/Unix
```

4. Compile and link by typing

```
make gauss
```

5. The BACC Gauss files are found in

```
cd Bacc/Compiled/Unix/Gauss
```

6. Make a note of the *absolute path* of the BACC Gauss Library and the BACC Dynamic Link Library. These have *relative paths* of

```
Bacc/Compiled/Unix/Gauss/libUnixBACC.lcg
```

and

```
Bacc/Compiled/Unix/Gauss/libBACC.so
```

respectively. Change directories to

```
Bacc/Compiled/Unix/Gauss
```

and type

```
pwd
```

at the Unix prompt. This will report the absolute path.

7. Configure the BACC Gauss Library: Edit the file `libUnixBACC.lcg` and change the absolute path of each entry to their new absolute paths.
8. Configure the BACC Dynamic Link Library: Edit the file `initUnixBACC.src` and change the absolute path of the file `libBACC.so` in the `dlibrary` call to its new absolute path.

Chapter 2

Models

2.1 Introduction

This document specifies the models currently supported by the BACC system. Each section following this one describes one of the supported models. Each model description is organized into subsections, following the pattern of this section. Appendix A gives the probability density and mass functions of the distributions used throughout the document.

2.1.1 Dimension parameters

All the quantities relevant to a model are treated as matrix valued. All matrix sizes are specified in terms of these dimension parameters. Examples of dimension parameters include the number of times a variable is observed, the number of individuals in a cross section, and the number of equations in a linear model. This subsection lists and describes the dimension parameters for a particular model.

2.1.2 Unknown Quantities

Unknown quantities are all the unobserved elements in a model. They include unknown parameters of the model, latent variables, and missing data. Separate sub-sub-sections discuss unknown quantities in each of these categories. Posterior simulation involves drawing these quantities from their posterior distribution; that is, their conditional distribution given known quantities.

2.1.3 Known Quantities

Known quantities are all the observed or user-specified values in a model. They include prior parameters, which index distributions within a family of prior distributions, and observed data. Separate sub-sub-sections discuss known quantities in both of these categories. The user of the BACC software must specify all the known quantities of a model in order to create an instance of the model.

2.1.4 Data Generating Process

This section specifies the conditional distribution of the endogenous observed data, given the unknown quantities and any observed data ancillary with respect to the unknown quantities.

2.1.5 Prior Distribution

This section specifies the marginal distribution of the unknown quantities, reflecting the user's prior beliefs about these quantities. These unknown quantities may or may not be independent. An example where they are not is a hierarchical prior, in which the prior density is expressed as the product of marginal densities of the "lowest level" unknowns and conditional densities of "higher level" unknowns given "lower level" unknowns.

2.1.6 Creating a Model Instance

This section gives all the model specific information a user requires to create a model instance. It specifies a short mnemonic label that identifies the model, the order in which the user gives the names to assign the unknown quantities, and the order in which to supply all the known quantities. To create a model instance, the user issues the `minst` command, with appropriate arguments (see section 3.4.14).

2.1.7 Sampling Algorithms

This section has brief descriptions of the algorithm used to generate samples of unknown quantities from their prior and posterior distributions. One subsection each concerns the prior distribution and the posterior distribution. For further details on the algorithms, the user should consult the internal (source code) documentation for the BACC system.

2.1.8 Marginal Likelihood

Where there is an analytical expression for the marginal likelihood in a model, this subsection provides that expression.

2.2 The Normal Linear Model

2.2.1 Dimension parameters

There are m equations, k covariate coefficients, and T observations of each variable.

2.2.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β and a $m \times m$ precision parameter H .

2.2.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient precision matrix \underline{H} , a precision degrees of freedom parameter $\underline{\nu} > \frac{m-1}{2}$, and a positive definite precision inverse scale parameter \underline{S} .

Data

There are m vectors of observations of dependent variables: y_1, \dots, y_m . Each vector is $T \times 1$.

There are m matrices of observations of ancillary (with respect to unknown quantities) variables: Z_1, \dots, Z_m . Each matrix is $T \times k$.

2.2.4 Data Generating Process

$$y \equiv \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} Z_1 \\ \vdots \\ Z_m \end{bmatrix} \beta + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_m \end{bmatrix} \equiv Z\beta + \epsilon$$

$$\begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_m \end{bmatrix} \mid \begin{bmatrix} Z_1 \\ \vdots \\ Z_m \end{bmatrix} \sim N(0, H^{-1} \otimes I_T)$$

2.2.5 Priors

The unknown parameters β and H are a-priori independent, and have the following marginal distributions.

$$\beta \sim N(\underline{\beta}, \underline{H}_\beta^{-1})$$

$$H \sim Wi(\underline{S}^{-1}, \underline{\nu})$$

When $m = 1$, the distribution of $\underline{S}H$ is chi-squared with $\underline{\nu}$ degrees of freedom.

2.2.6 Creating a Model Instance

The mnemonic label identifying the model is `nlm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example) and then the name of H .

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , $\underline{\nu}$, \underline{S} , Z , y .

2.2.7 Sampling Algorithms

Generating Prior Samples

Samples from the prior distribution of β and H are generated independently.

Generating Posterior Samples

The algorithm to generate samples from the posterior distribution $\beta, H|Z, y$ is a Gibbs sampling algorithm with two blocks, based on the following conditional posterior distributions.

$$\beta|H, y, Z \sim \text{N}(\bar{\beta}, \bar{H}_\beta^{-1})$$

$$H|\beta, y, Z \sim \text{W}(\bar{S}^{-1}, \bar{\nu})$$

where

$$\bar{H}_\beta = \underline{H}_\beta + Z'(H \otimes I_T)Z$$

$$\bar{\beta} = \bar{H}_\beta^{-1}[\underline{H}_\beta \beta + Z'(H \otimes I_T)y]$$

$$\bar{S} = \underline{S} + [s_{ij}], s_{ij} = (y_i - Z_i \beta)'(y_i - Z_i \beta)$$

$$\bar{\nu} = \underline{\nu} + T$$

2.3 The Seemingly Unrelated Regressions Model

This is a special case of the Normal Linear Model with $m > 1$. Please see section 2.2.

2.4 The I.I.D. Finite State Model

2.4.1 Dimension parameters

There are m states, N individuals and T observation times.

2.4.2 Unknown Quantities

Unknown Parameters

There is a $1 \times m$ state probability vector π .

2.4.3 Known Quantities

Prior Parameters

There is a $1 \times m$ parameter $\underline{\alpha}$ indexing the prior distribution of π .

Data

There are state observations $s_{ti} \in \{1, \dots, m\}$ for each individual i of N individuals and each observation period t of T periods.

$$S = \begin{bmatrix} s_{1,1} & \cdots & s_{N,1} \\ \vdots & \ddots & \vdots \\ s_{T,1} & \cdots & s_{N,T} \end{bmatrix}$$

2.4.4 Data Generating Process

Each observation s_{ti} is independently and identically distributed as follows.

$$\Pr(s_{ti} = s) = \pi_s \quad s = 1, \dots, m$$

2.4.5 Priors

$$\pi \sim \text{Di}(\underline{\alpha})$$

2.4.6 Creating a Model Instance

The mnemonic label identifying the model is `iidfs`.

Supply the name you wish to give the unknown quantity π (“pi” for example).

Supply the known quantities in the following order: $\underline{\alpha}$, S .

2.4.7 Sampling Algorithms

Generating Prior Samples

Samples from the prior distribution of π are generated independently.

Generating Posterior Samples

In this model, the posterior distribution for π is the following familiar distribution.

$$\pi|S \sim \text{Di}(\bar{\alpha})$$

where

$$\bar{\alpha} = \underline{\alpha} + n$$

$$n = [n_1 \cdots n_m]$$

and n_s is the number of observations for which $s_{ti} = s$. Posterior samples are drawn independently from this distribution.

2.4.8 Marginal Likelihood

The marginal likelihood is given by

$$p(S) = \frac{\Gamma(\sum_{i=1}^m \alpha_i) \prod_{i=1}^m \Gamma(\bar{\alpha}_i)}{\prod_{i=1}^m \Gamma(\alpha_i) \Gamma(\sum_{i=1}^m \bar{\alpha}_i)}.$$

2.5 The Non-Stationary First Order Markov Finite State Model

2.5.1 Dimension parameters

There are m states, N individuals and T observation times.

2.5.2 Unknown Quantities

Unknown Parameters

There is a $1 \times m$ initial state probability vector π and an $m \times m$ Markov transition probability matrix P .

2.5.3 Known Quantities

Prior Parameters

The prior parameters are a $1 \times m$ vector $\underline{\alpha}_0$ indexing the prior distribution of π , and an $m \times m$ matrix $\underline{\alpha}$ indexing the prior distribution of P .

Data

There are state observations $s_{ti} \in \{1, \dots, m\}$ for each individual i and each observation time t .

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TN} \end{bmatrix}$$

2.5.4 Data Generating Process

The N observation sequences $\{s_{ti}\}_{t=1}^T$ are i.i.d., with each sequence being first order Markov. The initial distribution is π and the Markov transition matrix is P .

$$\Pr(s_{1i} = s) = \pi_s \quad s = 1, \dots, m$$

$$\Pr(s_{ti} = s' | s_{t-1,i} = s) = P_{ss'}$$

2.5.5 Priors

The m rows P_s of P and π are mutually independent, and have the following marginal distributions.

$$\begin{aligned} \pi &\sim \text{Di}(\underline{\alpha}_0) \\ P_s &\equiv [P_{s1}, \dots, P_{sm}] \sim \text{Di}(\underline{\alpha}_s) \quad s = 1, \dots, m \\ \underline{\alpha}_0 &\equiv [\alpha_{01} \quad \cdots \quad \alpha_{0m}] \end{aligned}$$

$$\underline{\alpha}_s \equiv [\underline{\alpha}_{s1} \quad \cdots \quad \underline{\alpha}_{sm}] \quad s = 1, \dots, m$$

2.5.6 Creating a Model Instance

The mnemonic label identifying the model is `nsfomfs`.

Supply the names you wish to give the unknown quantities in the following order: first the name of π (“pi” for example), and then the name of P .

Supply the known quantities in the following order: $\underline{\alpha}_0$, $\underline{\alpha}$, S .

2.5.7 Sampling Algorithms

Generating Prior Samples

Samples from the prior distributions of π and P are generated independently.

Generating Posterior Samples

In the posterior distribution $\pi, P|S$, the parameters π and P are conditionally independent, and their marginal posterior distributions are the following familiar distributions.

$$\pi|S \sim \text{Di}(\bar{\alpha}_0)$$

$$P_s|S \sim \text{Di}(\bar{\alpha}_s) \quad s = 1, \dots, m$$

where

$$\begin{aligned} \bar{\alpha}_0 &\equiv \underline{\alpha}_0 + n_0 \\ \bar{\alpha} &\equiv \underline{\alpha} + n \\ \bar{\alpha}_s &\equiv [\bar{\alpha}_{s1} \quad \cdots \quad \bar{\alpha}_{sm}] \\ n_0 &\equiv [n_{01} \quad \cdots \quad n_{0m}] \\ n &\equiv \begin{bmatrix} n_{11} & \cdots & n_{1m} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mm} \end{bmatrix} \end{aligned}$$

where n_{0s} is the number of individuals starting in state s , and $n_{ss'}$ is the number of transitions from state s to state s' in the data.

Posterior samples are drawn independently from this distribution.

2.5.8 Marginal Likelihood

The marginal likelihood is available in closed form:

$$p(S) = \frac{\Gamma(\sum_{s=1}^m \underline{\alpha}_{0s}) \prod_{s=1}^m \Gamma(\bar{\alpha}_{0s})}{\prod_{s=1}^m \Gamma(\underline{\alpha}_{0s}) \Gamma(\sum_{s=1}^m \bar{\alpha}_{0s})} \cdot \prod_{s=1}^m \left[\frac{\Gamma(\sum_{s'=1}^m \underline{\alpha}_{ss'}) \prod_{s'=1}^m \Gamma(\bar{\alpha}_{ss'})}{\prod_{s'=1}^m \Gamma(\underline{\alpha}_{ss'}) \Gamma(\sum_{s'=1}^m \bar{\alpha}_{ss'})} \right]$$

2.6 The Stationary First Order Markov Finite State Model

2.6.1 Dimension parameters

There are m states, N individuals and T observation times.

2.6.2 Unknown Quantities

Unknown Parameters

There is an $m \times m$ Markov transition probability matrix P .

2.6.3 Known Quantities

Prior Parameters

The prior parameter is an $m \times m$ matrix $\underline{\alpha}$ indexing the prior distribution of P .

Data

There are state observations $s_{ti} \in \{1, \dots, m\}$ for each individual i and each observation time t .

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TN} \end{bmatrix}$$

2.6.4 Data Generating Process

The N observation sequences $\{s_{ti}\}_{t=1}^T$ are i.i.d., with each sequence being first order Markov with transition matrix P . The initial distribution vector is assumed to be the invariant distribution π for P .

$$\Pr(s_{1i} = s) = \pi_s \quad s = 1, \dots, m$$

$$\Pr(s_{ti} = s' | s_{t-1,i} = s) = P_{ss'}$$

where π is the left eigenvector of P corresponding to the eigenvalue $\lambda = 1$.

2.6.5 Priors

The m rows P_s of P are mutually independent, and have the following marginal distributions.

$$P_s \equiv [P_{s1}, \dots, P_{sm}] \sim \text{Di}(\underline{\alpha}_s) \quad s = 1, \dots, m$$

$$\underline{\alpha}_s \equiv [\underline{\alpha}_{s1} \quad \cdots \quad \underline{\alpha}_{sm}] \quad s = 1, \dots, m$$

2.6.6 Creating a Model Instance

The mnemonic label identifying the model is `sfomfs`.

Supply the name you wish to give the unknown quantity P .

Supply the known quantities in the following order: $\underline{\alpha}$, S .

2.6.7 Sampling Algorithms

Generating Prior Draws

Samples from the prior distribution of P are generated independently.

Generating Posterior Draws

In this model, an independence Metropolis-Hastings chain is used to draw from the posterior distribution for P . The distribution $P^*|S$ of candidate draws is

$$P_s^*|S \sim \text{Di}(\bar{\alpha}_s) \quad s = 1, \dots, m$$

where

$$\begin{aligned} \bar{\alpha} &\equiv \underline{\alpha} + n \\ \bar{\alpha}_s &\equiv [\bar{\alpha}_{s1} \quad \cdots \quad \bar{\alpha}_{sm}] \\ n &\equiv \begin{bmatrix} n_{11} & \cdots & n_{1m} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mm} \end{bmatrix} \end{aligned}$$

where $n_{ss'}$ is the number of transitions from state s to state s' in the data.

The Hastings ratio for this block is given by

$$\prod_{i=1}^N \frac{\pi_{S_{1i}}^*}{\pi_{S_{1i}}}$$

2.7 The Poisson Model

2.7.1 Dimension parameters

There are N observations.

2.7.2 Unknown Quantities

Unknown Parameters

There is a scalar mean parameter λ .

2.7.3 Known Quantities

Prior Parameters

There is a scalar shape parameter $\underline{\alpha} > 0$ and a scalar scale parameter $\underline{\beta} > 0$ indexing the prior distribution of λ .

Data

Each observation x_i is a non-negative integer.

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

2.7.4 Data Generating Process

The observations x_i are independently and identically Poisson distributed.

$$x_i \sim \text{Po}(\lambda)$$

2.7.5 Priors

$$\lambda \sim \text{Ga}(\underline{\alpha}, \underline{\beta})$$

2.7.6 Creating a Model Instance

The mnemonic label identifying the model is `poisson`

Supply the names you wish to give the unknown quantity λ (“lambda” for example).

Supply the known quantities in the following order: $\underline{\alpha}$, $\underline{\beta}$, X .

2.7.7 Sampling Algorithms

Generating Prior Samples

Samples from the prior distribution of λ are generated independently.

Generating Posterior Samples

In this model, the posterior distribution for λ is the following familiar distribution.

$$\lambda|X \sim \text{Ga}(\bar{\alpha}, \bar{\beta})$$

$$\bar{\alpha} = \underline{\alpha} + \sum_{i=1}^N x_i$$

$$\bar{\beta} = \underline{\beta} + N$$

Posterior samples are drawn independently from this distribution.

2.7.8 Marginal Likelihood

The marginal likelihood is given by the following expression.

$$p(X) = \frac{\underline{\beta}^N}{(\underline{\beta} + N)^{\underline{\alpha} + r}} \frac{\Gamma(\underline{\alpha} + r)}{\Gamma(\underline{\alpha})} \frac{1}{\prod_{i=1}^N x_i!},$$

where

$$r = \sum_{i=1}^N x_i$$

2.8 The Uniform Model

2.8.1 Dimension parameters

There are N observations.

2.8.2 Unknown Quantities

Unknown Parameters

There is a scalar support parameter θ .

2.8.3 Known Quantities

Prior Parameters

There is a scalar notional count parameter $\underline{\alpha} > 0$ and a scalar notional maximal element parameter $\underline{\beta} \geq 0$ indexing the prior distribution of θ .

Data

Each observation x_i is a non-negative real-valued scalar.

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

2.8.4 Data Generating Process

Each observation x is independently and identically distributed with a uniform distribution on $[0, \theta]$.

$$x_i \sim \text{i.i.d.} U(0, \theta)$$

2.8.5 Priors

$$\theta \sim \text{Pa}(\underline{\alpha}, \underline{\beta})$$

2.8.6 Creating a Model Instance

The mnemonic label identifying the model is **uniform**. Supply the name you wish to give the unknown quantity θ (“theta” for example).

Supply the known quantities in the following order: $\underline{\alpha}$, $\underline{\beta}$, X .

2.8.7 Sampling Algorithms

Generating Prior Samples

Samples from the prior distribution of θ are generated independently.

Generating Posterior Samples

In this model, the posterior distribution for θ is the following familiar distribution.

$$\theta|X \sim Pa(\bar{\alpha}, \bar{\beta})$$

where

$$\begin{aligned}\bar{\alpha} &= \underline{\alpha} + N \\ \bar{\beta} &= \max \left\{ \underline{\beta}, \max_i x_i \right\}\end{aligned}$$

Posterior samples are drawn independently from this distribution.

2.8.8 Marginal Likelihood

The marginal likelihood is given by the following expression.

$$p(X) = (\underline{\alpha}/\bar{\alpha}) \prod_{i=1}^N (p_i/\bar{\beta}_i)$$

where

$$\bar{\beta}_i = \max \{ \underline{\beta}, \max_{j \leq i} x_j \}$$

and

$$p_i = \begin{cases} (\frac{\bar{\beta}_i}{x_i})^{(1+\underline{\alpha}+i)} & \text{if } x_i > \bar{\beta}_i \\ 1 & \text{otherwise.} \end{cases}$$

2.9 A Univariate Linear Model with Normal Disturbances

The mnemonic label identifying the model is `n_u1m`.

Dimension Parameters

T number of observations

K number of covariates

Unknown Quantities

β ($K \times 1$) vector of covariate coefficients

h (1×1) precision of disturbance

Known Quantities

$\underline{\beta}$ ($K \times 1$) prior mean of β

\underline{H}_β ($K \times K$) prior precision of β

\underline{s}^2 (1×1) prior inverse scale of h

$\underline{\nu}$ (1×1) prior degrees of freedom of h

X ($T \times K$) covariates

y ($T \times 1$) dependant variable

Data Generating Process

The observables y are given by

$$y = X\beta + u,$$

where u is a $T \times 1$ vector of i.i.d. normal disturbances, with $u_t|h \sim N(0, h^{-1})$:

$$p(u|X, \beta, h) = (2\pi)^{-T/2} h^{T/2} \exp(-hu'u/2).$$

Prior Distribution

The vectors β and h , together with X , are mutually independent. The covariate coefficient vector β has distribution $N(\underline{\beta}, \underline{H}_\beta)$:

$$p(\beta) = (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta})/2].$$

The precision parameter h has a scaled chi-squared distribution, with $\underline{s}^2 h \sim \underline{\nu}$:

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h/2).$$

Sampling Algorithm

See Example 3.4.1 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.10 The Dichotomous Choice Model (with normally distributed disturbances)

2.10.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.10.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , and a $T \times 1$ vector \tilde{y} of latent outcomes.

2.10.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, and a positive definite precision inverse scale parameter \underline{S} .

Data

There is a $T \times 1$ vector of observations of dependent variables y taking values in $\{0, 1\}$.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.10.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.10.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.10.6 Creating a Model Instance

The mnemonic label identifying the model is `udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), and finally the name of the latent variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, X , y .

2.10.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

2.11 The Censored Linear Model (with normally distributed disturbances)

2.11.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.11.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , and a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes.

2.11.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , and a censoring parameter \underline{c} .

Data

There is a $T \times 1$ vector of observations of dependent variables y .

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.11.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.11.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.11.6 Creating a Model Instance

The mnemonic label identifying the model is **ucensor**.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), and finally the name of the latent variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, \underline{c} , X , y .

2.11.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.12 The Univariate Latent Linear Model (with normally distributed disturbances)

2.12.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.12.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , and a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes.

2.12.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, and a positive definite precision inverse scale parameter \underline{S} .

Data

Corresponding to the (possibly) latent outcome \tilde{y} , there are two $T \times 1$ vectors c and d , $c \geq d$, which describe the observed, set-valued outcome.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.12.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

2.12.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

2.12.6 Creating a Model Instance

The mnemonic label identifying the model is `ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), and finally the name of the latent variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, X , c , d .

2.12.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> .

2.13 A Univariate Linear Model with Student t Disturbances

The mnemonic label identifying the model is `t_ultm`.

Dimension Parameters

T number of observations

K number of covariates

Unknown Quantities

β ($K \times 1$) vector of covariate coefficients

h (1×1) precision of Student t distribution

\tilde{h} ($T \times 1$) time varying latent precision variable

λ (1×1) degrees of freedom of Student t distribution

Known Quantities

$\underline{\beta}$ ($K \times 1$) prior mean of β

\underline{H}_β ($K \times K$) prior precision of β

\underline{s}^2 (1×1) prior inverse scale of h

$\underline{\nu}$ (1×1) prior degrees of freedom of h

$\underline{\lambda}$ (1×1) prior mean of λ

X ($T \times K$) covariates

y ($T \times 1$) dependant variable

Data Generating Process

The observables y are given by

$$y = X\beta + u,$$

where u is a $T \times 1$ vector of independant Student t disturbances, with $u_t|h, X \sim t(0, h^{-1}, \lambda)$. Conditioning on the latent \tilde{h} gives $u_t|h, \tilde{h} \sim N(0, (h\tilde{h}_t)^{-1})$:

$$p(u|X, \beta, \tilde{h}, h) = (2\pi)^{-T/2} h^{T/2} \prod_{t=1}^T \tilde{h}_t^{1/2} \exp(-h\tilde{h}_t u_t^2/2).$$

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> for details.

Prior Distribution

The vectors β , h , and (\tilde{h}, λ) , together with X , are mutually independent. The covariate coefficient vector β has distribution $N(\underline{\beta}, \underline{H}_\beta)$:

$$p(\beta) = (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta})/2].$$

The precision parameter h has a scaled chi-squared distribution, with $\underline{s}^2 h \sim \underline{\nu}$:

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h/2).$$

The time varying latent precision parameters \tilde{h} are i.i.d. scaled chi-squared variates, with $\lambda \tilde{h}_t \sim \chi^2(\lambda)$:

$$p(\tilde{h}|\lambda) = [2^{\lambda/2} \Gamma(\lambda/2)]^{-T} \lambda^{T\lambda/2} \prod_{t=1}^T \tilde{h}_t^{(\lambda-2)/2} \exp(-\lambda \tilde{h}_t/2)$$

The degrees of freedom parameter λ is distributed $\exp(\underline{\lambda})$:

$$p(\lambda) = \underline{\lambda}^{-1} \exp(-\lambda/\underline{\lambda}).$$

Sampling Algorithm

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.14 The Dichotomous Choice Model (with Student t distributed disturbances)

2.14.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.14.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , a $T \times 1$ vector of precision parameters h_t , a $T \times 1$ vector \tilde{y} of latent outcomes, and a scalar degrees of freedom parameter λ .

2.14.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , and a degrees of freedom parameter $\underline{\lambda}$.

Data

There is a $T \times 1$ vector of observations of dependent variables y taking values in $\{0, 1\}$.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.14.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.14.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.14.6 Creating a Model Instance

The mnemonic label identifying the model is `t_udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), then the name of h_t (“hHetero” for example), then the name of the latent variable \tilde{y} (“yTilde” for example), and finally the name of the degrees of freedom parameter λ (“lambda” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, $\underline{\lambda}$, X , y .

2.14.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.15 The Censored Linear Model (with Student t distributed disturbances)

2.15.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.15.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , a $T \times 1$ vector of precision parameters h_t , a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes, and a scalar degrees of freedom parameter λ .

2.15.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , a degrees of freedom parameter $\underline{\lambda}$, and a censoring parameter \underline{c} .

Data

There is a $T \times 1$ vector of observations of dependent variables y .

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.15.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.15.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.15.6 Creating a Model Instance

The mnemonic label identifying the model is `t_ucensor`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), then the name of h_t (“hHetero” for example), then the name of the latent variable \tilde{y} (“yTilde” for example), and finally the name of the degrees of freedom parameter λ (“lambda” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, $\underline{\lambda}$, \underline{c} , X , y .

2.15.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.16 The Univariate Latent Linear Model (with Student t distributed disturbances)

2.16.1 Dimension parameters

There are k covariate coefficients and T observations of each variable.

2.16.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter β , a scalar precision parameter h , a $T \times 1$ vector of precision parameters h_t , a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes, and a scalar degrees of freedom parameter λ .

2.16.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , and a degrees of freedom parameter $\underline{\lambda}$.

Data

Corresponding to the (possibly) latent outcome \tilde{y} , there are two $T \times 1$ vectors c and d , $c \geq d$, which describe the observed, set-valued outcome.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.16.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.16.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.16.6 Creating a Model Instance

The mnemonic label identifying the model is `t_ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of β (“beta” for example), then the name of h (“hHomo” for example), then the name of h_t (“hHetero” for example), then the name of the latent variable \tilde{y} (“yTilde” for example), and finally the name of the degrees of freedom parameter λ (“lambda” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_β , \underline{S} , $\underline{\nu}$, $\underline{\lambda}$, X , c , d .

2.16.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.17 A Univariate Linear Model with Finite Mixtures of Normals Disturbances

The mnemonic label identifying the model is `fmm_ulm`.

Dimension Parameters

- T number of observations
 K number of covariates
 m number of mixture components (or states)

Unknown Quantities

- γ $((m + K) \times 1)$ vector of state means and covariate coefficients
 h (1×1) constant multiplicative precision component
 \mathbf{h} $(m \times 1)$ state dependant multiplicative precision component
 \tilde{s} $(T \times 1)$ time varying latent discrete state
 π $(1 \times m)$ state probabilities

Known Quantities

- \underline{h}_α (1×1) prior precision parameter for state means
 $\underline{\beta}$ $(K \times 1)$ prior mean of covariate coefficients
 \underline{H}_β $(K \times K)$ prior precision of covariate coefficients
 \underline{s}^2 (1×1) prior inverse scale of h
 $\underline{\nu}$ (1×1) prior degrees of freedom of h
 m (1×1) number of states
 $\underline{\nu}$ (1×1) degrees of freedom parameter for state precisions
 r (1×1) Dirichlet parameter for state probabilities
 X $(T \times K)$ covariates
 y $(T \times 1)$ dependant variable

Data Generating Process

The observables y are given by

$$y = X\beta + u,$$

where u is a $T \times 1$ vector of independent discrete normal mixture disturbances, with $u_t|h, \pi, \alpha, \mathbf{h}, X$ given by:

$$p(u_t|h, \pi, \alpha, \mathbf{h}, X) = (2\pi)^{-1/2} h^{1/2} \sum_{j=1}^m \pi_j h_j^{1/2} \exp[-h \cdot h_j (u_t - \alpha_j)^2 / 2]$$

Conditioning on the latent states gives

$$p(u_t|h, \alpha, \mathbf{h}, X) = (2\pi)^{-1/2} h^{1/2} h_{\bar{s}_t}^{1/2} \exp[-h \cdot h_{\bar{s}_t} (u_t - \alpha_{\bar{s}_t})^2 / 2].$$

See Section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html> for details.

Prior Distribution

The vectors (γ, h) , \mathbf{h} , and (\bar{s}, π) , together with X , are mutually independent. The γ parameter vertically stacks the parameters α and β , where α is the $m \times 1$ vector of state dependant means and β is the $K \times 1$ vector of covariate coefficients. They are independent, with $\alpha|h \sim N(0, (\underline{h}_\alpha h)^{-1})$ and $\beta \sim N(\underline{\beta}, \underline{H}_\beta)$:

$$\begin{aligned} p(\gamma|h) = p(\alpha|h) \cdot p(\beta) &= (2\pi)^{-m/2} (\underline{h}_\alpha h)^{m/2} \exp(-\underline{h}_\alpha h \alpha' \alpha / 2) \\ &\cdot (2\pi)^{-K/2} |\underline{H}_\beta|^{1/2} \exp[-(\beta - \underline{\beta})' \underline{H}_\beta (\beta - \underline{\beta}) / 2]. \end{aligned}$$

The precision parameter h has a scaled chi-squared distribution, with $\underline{s}^2 h \sim \underline{\nu}$:

$$p(h) = 2^{-\underline{\nu}/2} \Gamma(\underline{\nu}/2)^{-1} (\underline{s}^2)^{\underline{\nu}/2} h^{(\underline{\nu}-2)/2} \exp(-\underline{s}^2 h / 2).$$

The state dependant precisions h_j are i.i.d., with $\underline{\nu} \cdot h_j \sim \chi^2(\underline{\nu})$:

$$p(\mathbf{h}) = 2^{m\underline{\nu}} \Gamma(\underline{\nu}/2) (\underline{\nu}^2)^{m\underline{\nu}/2} \prod_{j=1}^m h_j^{(-\underline{\nu}-2)/2} \exp(-\underline{\nu} \cdot h_j / 2).$$

The latent states are i.i.d., with the probability $\Pr[s_t = j]$ given by π_j , for $j = 1, \dots, m$:

$$p(\bar{s}|\pi) = \prod_{t=1}^T \pi_{\bar{s}_t}$$

The vector π of probabilities is distributed Dirichlet(r, \dots, r):

$$p(\pi) = \Gamma(mr) \Gamma(r)^{-m} \prod_{j=1}^m \pi_j^{r-1}.$$

Sampling Algorithm

See section 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.18 The Dichotomous Choice Model (with a scale mixture of normals distribution for the disturbances)

2.18.1 Dimension parameters

There are k covariate coefficients, T observations of each variable, and m components for the mixture of normals (i.e., m states).

2.18.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter γ , a scalar precision parameter h , a $T \times 1$ vector of state indices, a $1 \times m$ vector of probabilities, an $m \times 1$ vector of precision parameters h_j , and a $T \times 1$ vector \tilde{y} of latent outcomes.

2.18.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a positive scalar precision parameter \underline{H}_α , a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , an $m \times 1$ vector of precision degrees of freedom parameters $\underline{\nu}_j$, an $m \times 1$ vector of positive definite precision inverse scale parameter \underline{S}_j , and a $1 \times m$ vector of hyperparameters $\underline{\rho}$.

Data

There is a $T \times 1$ vector of observations of dependent variables y taking values in $\{0, 1\}$.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.18.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.18.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.18.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.udcht`.

Supply the names you wish to give the unknown quantities in the following order: first the name of γ (“gamma” for example), then the name of s , then the name of p , then the name of h_j (“hState” for example), and finally the name of the latent outcome variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_α , \underline{H}_β , \underline{S} , $\underline{\nu}$, \underline{S}_j , $\underline{\nu}_j$, \underline{r} , X , y .

2.18.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.19 The Censored Linear Model (with a scale mixture of normals distribution for the disturbances)

2.19.1 Dimension parameters

There are k covariate coefficients, T observations of each variable, and m components for the mixture of normals (i.e., m states).

2.19.2 Unknown Quantities

Unknown Parameters

There is a $k \times 1$ coefficient parameter γ , a scalar precision parameter h , a $T \times 1$ vector of state indices, a $1 \times m$ vector of probabilities, an $m \times 1$ vector of precision parameters h_j , and a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes.

2.19.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a positive scalar precision parameter \underline{H}_α , a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , an $m \times 1$ vector of precision degrees of freedom parameters $\underline{\nu}_j$, an $m \times 1$ vector of positive definite precision inverse scale parameter \underline{S}_j , a $1 \times m$ vector of hyperparameters $\underline{\tau}$, and a censoring parameter \underline{c} .

Data

There is a $T \times 1$ vector of observations of dependent variables y .

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.19.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.19.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.19.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.ucensor`.

Supply the names you wish to give the unknown quantities in the following order: first the name of γ (“gamma” for example), then the name of s , then the name of p , then the name of h_j (“hState” for example), and finally the name of the outcome variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\beta}$, \underline{H}_α , \underline{H}_β , \underline{S} , $\underline{\nu}$, \underline{S}_j , $\underline{\nu}_j$, \underline{r} , \underline{c} , X , y .

2.19.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.20 The Univariate Latent Linear Model (with a scale mixture of normals distribution for the disturbances)

2.20.1 Dimension parameters

There are k covariate coefficients, T observations of each variable, and m components for the mixture of normals (i.e., m states).

2.20.2 Unknown Quantities

Unknown Parameters

There is a $(m+k) \times 1$ coefficient parameter γ , a scalar precision parameter h , a $T \times 1$ vector of state indices, a $1 \times m$ vector of probabilities, an $m \times 1$ vector of precision parameters h_j , and a $T \times 1$ vector \tilde{y} of (possibly) latent outcomes.

2.20.3 Known Quantities

Prior Parameters

There is a $k \times 1$ coefficient mean vector $\underline{\beta}$, a positive scalar precision parameter \underline{H}_α , a $k \times k$ positive definite coefficient matrix \underline{H}_β , a precision degrees of freedom parameter $\underline{\nu}$, a positive definite precision inverse scale parameter \underline{S} , an $m \times 1$ vector of precision degrees of freedom parameters $\underline{\nu}_j$, an $m \times 1$ vector of positive definite precision inverse scale parameter \underline{S}_j , and a $1 \times m$ vector of hyperparameters \underline{x} .

Data

Corresponding to the (possibly) latent outcome \tilde{y} , there are two $T \times 1$ vectors c and d , $c \geq d$, which describe the observed, set-valued outcome.

There is a $T \times k$ matrix X of observations of ancillary (with respect to unknown quantities) variables.

2.20.4 Data Generating Process

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.20.5 Priors

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.20.6 Creating a Model Instance

The mnemonic label identifying the model is `fmn.ullm`.

Supply the names you wish to give the unknown quantities in the following order: first the name of γ (“gamma” for example), then the name of s , then the name of p , then the name of h_j (“hState” for example), and finally the name of the outcome variable \tilde{y} (“yTilde” for example).

Supply the known quantities in the following order: $\underline{\gamma}$, \underline{H}_α , \underline{H}_β , \underline{S} , $\underline{\nu}$, \underline{S}_j , $\underline{\nu}_j$, \underline{r} , X , c , d .

2.20.7 Sampling Algorithms

See Sections 4.5 and 4.8 in “Contemporary Bayesian Econometrics and Statistics,” by John Geweke, at <http://www.cirano.qc.ca/~bacc/bacc2003/resources.html>

2.21 An Autoregression Model

2.21.1 Dimension Parameters

The model features the following dimension parameters.

Dimension Parameter	Description
T	number of observations
K	number of covariates
p	autoregressive order

2.21.2 Unknown Quantities

The model features the following unknown quantities.

Unknown Quantity	Dimensions	Description
β	$K \times 1$	covariate coefficient vector
h	1×1	residual precision
ϕ	$p \times 1$	vector of autoregression coefficients

2.21.3 Known Quantities

The model features the following known quantities.

Known Quantity	Dimensions	Description
$\bar{\beta}$	$K \times 1$	prior mean of β
\bar{H}_β	$K \times K$	prior precision of β
$\bar{\nu}$	1×1	prior degrees of freedom of h
\bar{s}^2	1×1	prior inverse scale of h
$\bar{\phi}$	$p \times 1$	prior mean of ϕ before truncation
\bar{H}_ϕ	$p \times p$	prior precision of ϕ before truncation
X	$T \times K$	covariates
y	$T \times 1$	dependant variable

2.21.4 Data Generating Process

The data generating process is given by

$$y_t = \beta' x_t + \epsilon_t$$

where x_t is the t 'th row of X , as a column vector,

$$\epsilon_t = \sum_{i=1}^p \phi_i \epsilon_{t-i} + u_t,$$

and

$$u_t \sim \text{i.i.d. } N(0, h^{-1})$$

2.21.5 Prior Distribution

The unknowns are a-priori independent and have the following distributions.

$$\beta \sim N(\bar{\beta}, \bar{H}_\beta^{-1})$$

$$\bar{s}^2 h \sim \chi^2(\bar{\nu})$$

The prior for ϕ is obtained by truncating the following density to the region for which y is stationary.

$$\phi \sim N(\bar{\phi}, \bar{H}_\phi^{-1})$$

2.21.6 Creating a Model Instance

The mnemonic label identifying the model is **AR**.

Supply the names you wish to give the unknown quantities in the same order as they appear in the table of unknown quantities. Supply the known quantities in the same order as they appear in the table of known quantities.

2.21.7 Sampling Algorithm

The sampling algorithm for prior simulation features three blocks, each making independent draws from the prior distribution of one of the unknown quantities. The sampling algorithm for posterior simulation features three blocks, each making draws from the conditional posterior distribution of one of the unknown quantities.

2.22 An Autoregression Model with State Dependant Means

2.22.1 Dimension Parameters

The model features the following dimension parameters.

Dimension Parameter	Description
T	number of observations
K	number of covariates
p	autoregressive order
m	number of states

2.22.2 Unknown Quantities

The model features the following unknown quantities.

Unknown Quantity	Dimensions	Description
γ	$(m + K) \times 1$	vertical stack of alpha and beta
h	1×1	residual precision
ϕ	$p \times 1$	vector of autoregression coefficients
P	$m \times m$	state transition probability matrix
s	$T \times 1$	latent states
f	$T \times m$	filter probabilities

2.22.3 Known Quantities

The model features the following known quantities.

Known Quantity	Dimensions	Description
$\bar{\gamma}$	$(m + K) \times 1$	prior mean of γ
\bar{H}_γ	$(m + K) \times (m + K)$	prior precision of γ
$\bar{\nu}$	1×1	prior degrees of freedom of h
\bar{s}^2	1×1	prior inverse scale of h
$\bar{\phi}$	$p \times 1$	prior mean of ϕ before truncation
\bar{H}_ϕ	$p \times p$	prior precision of ϕ before truncation
\bar{A}	$m \times m$	parameters of prior for P
X	$T \times K$	covariates
y	$T \times 1$	dependant variable

2.22.4 Data Generating Process

The data generating process is given by

$$y_t = \alpha_{s_t} + \beta' x_t + \epsilon_t$$

where x_t is the t 'th row of X , as a column vector, and α ($m \times 1$) and β ($K \times 1$) are obtained by partitioning γ ,

$$\epsilon_t = \sum_{i=1}^p \phi_i \epsilon_{t-i} + u_t,$$

and

$$u_t \sim \text{i.i.d. } N(0, h^{-1})$$

2.22.5 Prior Distribution

The unknowns are a-priori independent and have the following distributions.

$$\begin{aligned} \gamma &\sim N(\bar{\gamma}, \bar{H}_\gamma^{-1}) \\ \bar{s}^2 h &\sim \chi^2(\bar{\nu}) \end{aligned}$$

The prior for ϕ is obtained by truncating the following density to the region for which y is stationary.

$$\begin{aligned} \phi &\sim N(\bar{\phi}, \bar{H}_\phi^{-1}) \\ (P_{i1}, \dots, P_{im}) &\sim \text{i.i.d. Di}(\bar{A}_{i1}, \dots, \bar{A}_{im}) \\ \Pr[s_t = j | s_{t-1} = i] &= P_{ij} \end{aligned}$$

The unknown quantity f gives, for each observation time t , the state probabilities at t given previous states, previous values of the observed variables, and the other unknown quantities. It is not a primitive unknown quantity, and it is included to give the user access to filtered probabilities.

2.22.6 Creating a Model Instance

The mnemonic label identifying the model is `Hamilton`.

Supply the names you wish to give the unknown quantities in the same order as they appear in the table of unknown quantities. Supply the known quantities in the same order as they appear in the table of known quantities.

2.22.7 Sampling Algorithm

The sampling algorithm for prior simulation features five blocks. Four blocks make independent draws from the prior distributions of γ , h , ϕ and P . The fifth makes draws from the distribution $s|P$. The sampling algorithm for posterior simulation features five blocks, each making draws from the conditional posterior distribution of one of the unknown quantities.

Chapter 3

BACC Commands

3.1 Overview of BACC Commands

The following is a list of BACC commands with brief descriptions.

<code>dirichletSim</code>	Generates a sample from a multiple Dirichlet distribution.
<code>expect1</code>	Calculates, for a weighted random sample, the sample mean and standard deviation, estimates of the numerical standard error for the mean, and estimates of the relative numerical efficiency.
<code>expectN</code>	Calculates combined sample means, with numerical standard errors, for a set of different weighted random samples, and tests for the equality of their individual population means.
<code>extract</code>	Returns simulation matrices for a model instance.
<code>gammaSim</code>	Generates a sample from a gamma distribution.
<code>gaussianSim</code>	Generates a sample from a Gaussian distribution.
<code>listModelSpecs</code>	Lists all available model specifications (e.g. <code>nlm</code> , <code>poisson</code>).
<code>listModels</code>	Lists all open model instances.
<code>miDelete</code>	Closes without saving a (or all) model instances.
<code>miLoad</code>	Loads a model instance stored in a binary file.
<code>miLoadAscii</code>	Loads a model instance stored in a text file.
<code>miSave</code>	Saves a model instance in a binary file.
<code>miSaveAscii</code>	Saves a model instance in a text file.
<code>minst</code>	Creates an instance of a particular model specification.

<code>mlike</code>	Computes various estimates of the marginal likelihood for a model instance, with numerical standard errors.
<code>paretoSim</code>	Generates a sample from a Pareto distribution.
<code>postfilter</code>	Filters out previously generated draws from the posterior simulation matrix of a given model instance.
<code>postsim</code>	Generates or appends to the posterior simulation matrix of a given model instance.
<code>postsimHM</code>	Generates or appends to the posterior HM simulation matrix of a given model instance.
<code>priorRobust</code>	Calculates upper and lower bounds on the mean of a posterior function of interest, as the prior distribution is varied from its original specification.
<code>priorfilter</code>	Filters out previously generated draws from the prior simulation matrix of a given model instance.
<code>priorsim</code>	Generates or appends to the prior simulation matrix of a given model instance.
<code>setseedconstant</code>	Sets the seeds of the random number generators to a constant value.
<code>setseedtime</code>	Sets the seeds of the random number generators to the number of seconds since the beginning of 1970.
<code>weightedSmooth</code>	Estimates a univariate density function for a weighted random sample, using a kernel smoothing algorithm adapted to weighted samples.
<code>wishartSim</code>	Generates a sample from a Wishart distribution.

3.2 Miscellaneous Gauss Issues

3.2.1 Running BACC within Gauss

The commands in the BACC Gauss Library use functions in the BACC Dynamic Link Library. Therefore, each time you use BACC for Gauss you will need to set up both libraries before using any BACC commands. You can do this by entering the commands

```
library /path/libUnixBACC; /* set-up the BACC Gauss library */
initUnixBACC; /* set-up the BACC Dynamic Link Library */
```

at the Gauss prompt, where *path* is the absolute path¹ of the directory which contains the file `libUnixBACC.lcg`.

¹See item 6 in Section 1.3

3.2.2 BACC's use of Global Variables

BACC commands `expect1`, `expectN`, `mlike`, `robust` and `smooth` use global variables for optional parameters. To set the value of a parameter, set the appropriate global variable to the desired value. Where variables are not set, the parameters take on default values. Once these variables are set, they remain in the symbol table until they are deleted. To make a parameter revert to its default value, delete the corresponding global variable. See the examples in the command descriptions.

3.3 Miscellaneous Gauss Issues

3.3.1 Running BACC within Gauss

The commands in the BACC Gauss Library use functions in the BACC Dynamic Link Library. Therefore, each time you use BACC for Gauss you will need to set up both libraries before using any BACC commands. You can do this by entering the commands

```
library /path/libUnixBACC; /* set-up the BACC Gauss library */
initUnixBACC; /* set-up the BACC Dynamic Link Library */
```

at the Gauss prompt, where *path* is the absolute path² of the directory which contains the file `libUnixBACC.lcg`.

3.3.2 BACC's use of Global Variables

BACC commands `expect1`, `expectN`, `mlike`, `priorRobust` and `weightedSmooth` use global variables for optional parameters. To set the value of a parameter, set the appropriate global variable to the desired value. Where variables are not set, the parameters take on default values. Once these variables are set, they remain in the symbol table until they are deleted. To make a parameter revert to its default value, delete the corresponding global variable. See the examples in the command descriptions.

3.4 Detailed Description of Commands

Each BACC command is described in detail in one of the following sections.

²See item 6 in Section 1.3

3.4.1 The `dirichletSim` Command

Description

Generates a sample from a multiple Dirichlet distribution.

Usage

```
sample = dirichletSim(A, n);
```

Inputs

<code>A</code>	m by K matrix: Dirichlet parameters
<code>n</code>	Integer: number of draws to generate

Outputs

<code>sample</code>	n by nK matrix: sample generated from multiple Dirichlet distribution
---------------------	---

See Also

`paretoSim`, `gaussianSim`, `gammaSim`, `wishartSim`.

Example

```
A = 1.0 2.0 3.0,4.0 5.0 6.0;
sample = dirichletSim(A, 1000);
```

Details

The sample consists of n draws. Each of the n draws of the sample is an m by K matrix with independent rows. Each row has a Dirichlet distribution with parameters given by the corresponding row of A .

The result is given as a n by mK matrix, and each column gives a draw in column major order. See Appendix A for the parameterization of the Dirichlet distribution.

3.4.2 The `expect1` Command

Description

Calculates, for a weighted random sample, the sample mean and standard deviation, estimates of the numerical standard error for the mean, and estimates of the relative numerical efficiency.

Usage

```
mean, std, nse, rne = expect1(logWeight, sample);
```

Inputs

<code>logWeight</code>	Vector of length M : log sample weights
<code>sample</code>	Vector of length M : sample of scalar draws
<code>taper</code>	Vector of length K : taper half-widths (optional)

Outputs

<code>mean</code>	Real scalar: weighted sample mean
<code>std</code>	Real scalar: weighted sample standard deviation
<code>nse</code>	Vector of length $K + 1$: estimated numerical standard errors
<code>rne</code>	Vector of length $K + 1$: estimated relative numerical efficiency

See Also

`expectN`, `priorRobust`.

Example

```
/* Use default taper values */
mean, std, nse, rne = expect1(lw, z);
/* Use alternate taper values */
taper = 4.0 8.0;
mean, std, nse, rne = expectN(lw, z, taper);
```

Details

Let $z = (z_1, \dots, z_M)$ be the sample and $(\log w_1, \dots, \log w_M)$ be the vector of log weights. Let $\lambda = (\lambda_1, \dots, \lambda_K)$ be the vector of half-widths. The sample is broken into T groups of size $J \equiv M \text{ div } T$ and the last $M \bmod T$ elements are ignored. Thus $M_{\text{use}} \equiv JT$ elements are used.

The sample mean and standard deviation are calculated as follows:

$$\bar{z} = \frac{\sum_{m=1}^{M_{use}} w_m z_m}{\sum_{m=1}^{M_{use}} w_m}$$

$$\sigma_z = \left[\frac{\sum_{m=1}^{M_{use}} w_m (z_m - \bar{z})^2}{\sum_{m=1}^{M_{use}} w_m} \right]^{\frac{1}{2}}$$

For the calculation of the first numerical standard error τ_0 , we assume no serial correlation in (z_1, \dots, z_M) . This is appropriate for independence or importance sampling. Following Geweke (1989) [3], this leads to

$$\tau \approx \tau_0 \equiv \left[\frac{\sum_{m=1}^{M_{use}} w_m^2 (z_m - \bar{z})^2}{\left(\sum_{m=1}^{M_{use}} w_m \right)^2} \right]^{\frac{1}{2}}$$

For the calculation of τ_1 through τ_K , the remaining K estimates of the numerical standard error, the following method is used. First, `expect1` calculates group and sample means of the numerator quantity $w_m z_m$ and the denominator quantity w_m :

$$n(t) = \frac{1}{J} \sum_{m=(t-1)J+1}^{tJ} w_m z_m \quad d(t) = \frac{1}{J} \sum_{m=(t-1)J+1}^{tJ} w_m \quad t = 1, \dots, T$$

$$\bar{n} = \frac{1}{M_{use}} \sum_{m=1}^{M_{use}} w_m z_m \quad \bar{d} = \frac{1}{M_{use}} \sum_{m=1}^{M_{use}} w_m$$

Then it calculates the following sample autocorrelation and autocovariance functions:

$$\gamma_{nn}(t) = \frac{1}{T} \sum_{s=t+1}^T (n(s) - \bar{n})(n(s-t) - \bar{n}) \quad t = 0, \dots, T-1$$

$$\gamma_{dd}(t) = \frac{1}{T} \sum_{s=t+1}^T (d(s) - \bar{d})(d(s-t) - \bar{d}) \quad t = 0, \dots, T-1$$

$$\gamma_{nd}(t) = \frac{1}{T} \sum_{s=t+1}^T (n(s) - \bar{n})(d(s-t) - \bar{d}) \quad t = 0, \dots, T-1$$

$$\gamma_{dn}(t) = \frac{1}{T} \sum_{s=t+1}^T (d(s) - \bar{d})(n(s-t) - \bar{n}) \quad t = 0, \dots, T-1$$

Then it calculates, for each $k \in \{1, \dots, K\}$, estimates $\sigma_{n(k)}^2$, $\sigma_{d(k)}^2$, and $\sigma_{nd(k)}$ of $\sigma_n^2 \equiv \text{Var}[n(t)]$, $\sigma_d^2 \equiv \text{Var}[d(t)]$ and $\sigma_{nd} \equiv \text{Cov}[n(t), d(t)]$, based on the taper half-width λ_k :

$$\sigma_{nn(k)}^2 = \gamma_{nn}(0) + 2 \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} \gamma_{nn}(s)$$

$$\sigma_{dd(k)}^2 = \gamma_{dd}(0) + 2 \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} \gamma_{dd}(s)$$

$$\sigma_{nd(k)}^2 = \gamma_{nd}(0) + \sum_{s=1}^{\lambda_k-1} \frac{\lambda_k - s}{\lambda_k} [\gamma_{nd}(s) + \gamma_{dn}(s)]$$

These calculations are based on conventional time series methods for a wide sense stationary process, described in Geweke (1992) [4].

By the conventional asymptotic expansion, the square of the numerical standard error is approximated by

$$\tau^2 = \text{Var}\left(\frac{n}{d}\right) \approx \begin{bmatrix} \frac{1}{d} & -\frac{1}{d^2} \end{bmatrix} \begin{bmatrix} \sigma_n^2 & \sigma_{nd} \\ \sigma_{nd} & \sigma_d^2 \end{bmatrix} \begin{bmatrix} \frac{1}{d} \\ -\frac{1}{d^2} \end{bmatrix}$$

For each $k \in \{1, \dots, K\}$ it calculates the approximation τ_k using $\sigma_{nn(k)}^2$, $\sigma_{dd(k)}^2$ and $\sigma_{nd(k)}^2$ defined above.

Relative numerical efficiencies (ν_0, \dots, ν_K) are calculated using

$$\nu_k \equiv \left(\frac{\tau_0}{\tau_k}\right)^2 \quad k = 0, \dots, K$$

3.4.3 The expectN Command

Description

Calculates combined sample means, with numerical standard errors, for a set of different weighted random samples, and tests for the equality of their individual population means.

Usage

```
mean, nse, equal = expectN(logWeight1, sample1, logWeight2, sample2);
```

Inputs

<code>logWeight1</code>	Vector of length M_1 : log sample weights for first sample
<code>sample1</code>	Vector of length M_1 : first sample of scalar draws
<code>logWeight2</code>	Vector of length M_2 : log sample weights for second sample
<code>sample2</code>	Vector of length M_2 : second sample of scalar draws
<code>taper</code>	Vector of length K : taper half-widths (optional)

Outputs

<code>mean</code>	Vector of length $K + 1$: estimated combined weighted sample means
<code>nse</code>	Vector of length $K + 1$: estimated numerical standard errors
<code>equal</code>	Vector of length $K + 1$: marginal significance levels for a chi-squared test of the equality of the population means

See Also

`expect1`.

Example

```
/* Use default taper values */
mean, nse, p = expectN(lw1, z1, lw2, z2);
/* Use alternate taper values */
taper = 4.0 8.0;
mean, nse, p = expectN(lw1, z1, lw2, z2, taper);
```

Details

In general, there are N pairs of weighted samples, not just two. For each sample $z^{(i)}$, **expectN** calculates individual sample moments $\bar{z}^{(i)}$ and estimates of numerical standard errors $(\tau_0^{(i)}, \dots, \tau_K^{(i)})$ from the samples $(z_1^{(i)}, \dots, z_{M_i}^{(i)})$, the log weights $(\log z_1^{(i)}, \dots, \log z_{M_i}^{(i)})$, and the half-taper values $(\lambda_1, \dots, \lambda_K)$, in the same way that **expect1** calculates \bar{z} and (τ_0, \dots, τ_K) from (z_1, \dots, z_M) , $(\log z_1, \dots, \log z_M)$, and $(\lambda_1, \dots, \lambda_K)$.

The estimated sample means \bar{z}_k are given by

$$\bar{z}_k = \sum_{i=1}^N \frac{\bar{z}^{(i)}}{\tau_k^{2(i)}} \bigg/ \sum_{i=1}^N \frac{1}{\tau_k^{2(i)}} \quad k = 1, \dots, K$$

The estimated numerical standard errors τ_k are given by

$$\frac{1}{\tau_k^2} = \sum_{i=1}^N \frac{1}{\tau_k^{2(i)}}$$

For each k , the marginal significance level is the value of p_k such that

$$\begin{bmatrix} \bar{z}_k^{(2)} - \bar{z}_k^{(1)} & \dots & \bar{z}_k^{(N)} - \bar{z}_k^{(N-1)} \end{bmatrix} \cdot \Sigma^{-1} \cdot \begin{bmatrix} \bar{z}_k^{(2)} - \bar{z}_k^{(1)} \\ \bar{z}_k^{(3)} - \bar{z}_k^{(2)} \\ \vdots \\ \bar{z}_k^{(N)} - \bar{z}_k^{(N-1)} \end{bmatrix} = \chi_{1-p_k}^2(N-1)$$

where Σ is the following matrix

$$\begin{bmatrix} \tau_k^{2(1)} + \tau_k^{2(2)} & -\tau_k^{2(2)} & 0 & \dots & 0 & 0 \\ -\tau_k^{2(2)} & \tau_k^{2(2)} + \tau_k^{2(3)} & -\tau_k^{2(3)} & \dots & 0 & 0 \\ 0 & -\tau_k^{2(3)} & \tau_k^{2(3)} + \tau_k^{2(4)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \tau_k^{2(N-2)} + \tau_k^{2(N-1)} & -\tau_k^{2(N-1)} \\ 0 & 0 & 0 & \dots & -\tau_k^{2(N-1)} & \tau_k^{2(N-2)} + \tau_k^{2(N-1)} \end{bmatrix}$$

3.4.4 The extract Command

Description

Returns simulation matrices for a model instance.

Usage

```
sim = extract(modelInst);
```

Inputs

`modelInst` Integer: model instance identifier.

Outputs

`sim` Structure: simulation matrices

Example

```
sim = extract(mi);
```

Details

The return value is a structure (named list in S-PLUS and R) with the following fields (components in S-PLUS and R):

id Model instance identifier.

logWeightPost Log weights for posterior draws.

logPrior Value of log prior for posterior draws.

logXPrior Value of transformed log prior values for posterior draws

logLike Value of log likelihood for posterior draws.

logPriorHM Value of log prior for posterior HM draws.

logLikeHM Value of log likelihood for posterior HM draws.

logWeightPrior Log weights for prior draws.

logPriorPrior Value of log prior for prior draws.

logLikePrior Value of log likelihood for prior draws.

* Posterior simulation matrix of unknown quantity named *.

***Prior** Prior simulation matrix of unknown quantity named *.

***HM** Posterior HM simulation matrix of unknown quantity named *.

All simulation matrices have three dimensions. The first two dimensions give the row and column of the unknown quantity. The third dimension is the simulation dimension. Each value of the third index gives a different draw of the unknown quantity.

3.4.5 The gammaSim Command

Description

Generates a sample from a gamma distribution.

Usage

```
sample = gammaSim(alpha, beta, n);
```

Inputs

alpha	Real scalar: shape parameter of gamma distribution
beta	Real scalar: inverse scale parameter of gamma distribution
n	Integer: number of draws to generate

Outputs

sample	n by 1 matrix: sample generated from gamma distribution
--------	---

See Also

paretoSim, gaussianSim, dirichletSim, wishartSim.

Example

```
alpha = 3.0  
beta = 5.0  
sample = gammaSim(alpha, beta, 1000);
```

Details

Each of the n draws of the sample is a scalar with a gamma distribution. The result is given as a n by 1 matrix.

See Appendix A for the parametrization of the gamma distribution.

3.4.6 The gaussianSim Command

Description

Generates a sample from a Gaussian distribution.

Usage

```
sample = gaussianSim(mean, precision, n);
```

Inputs

mean	Vector of length K : mean of Gaussian distribution
precision	K by K matrix: precision of Gaussian distribution
n	Integer: number of draws to generate

Outputs

sample	n by K matrix: sample generated from Gaussian distribution
--------	--

See Also

paretoSim, dirichletSim, gammaSim, wishartSim.

Example

```
mean = 1.0,2.0;  
precision = 1.0 0.0,0.0 1.0;  
sample = gaussianSim(mean, precision, 1000);
```

Details

Each of the n draws of the sample is a vector of length K with a Gaussian distribution. The result is given as a n by K matrix.

See Appendix A for the parametrization of the Gaussian distribution.

3.4.7 The `listModelSpecs` Command

Description

Lists all available model specifications (e.g. `nlm`, `poisson`).

Usage

```
listModelSpecs;
```

Inputs

None.

Outputs

None.

See Also

`minst`, `listModels`.

Example

```
listModelSpecs;
```

Details

A printed message gives a list of model specifications.

3.4.8 The `listModels` Command

Description

Lists all open model instances.

Usage

```
listModels;
```

Inputs

None.

Outputs

None.

See Also

`minst`, `miDelete`, `listModelSpecs`.

Example

```
listModels;
```

Details

A printed message gives the model instance identification number, the name of the model specification (e.g. `nlm`, `poisson`), and the number of prior, posterior, and posterior HM draws.

3.4.9 The miDelete Command

Description

Closes without saving a (or all) model instances.

Usage

```
miDelete(modelInst);
```

Inputs

`modelInst` Integer: model instance identifier.

Outputs

None.

See Also

`minst`, `listModels`.

Example

```
miDelete(mi);
```

3.4.10 The miLoad Command

Description

Loads a model instance stored in a binary file.

Usage

```
modelInst = miLoad(filename);
```

Inputs

`filename` String: name of binary file storing the model instance

Outputs

`modelInst` Integer: model instance identifier.

See Also

`miSave`, `minst`, `miLoadAscii`.

Example

```
mi = miLoad("miFile");
```

3.4.11 The miLoadAscii Command

Description

Loads a model instance stored in a text file.

Usage

```
modelInst = miLoadAscii(filename);
```

Inputs

`filename` String: name of text file storing the model instance

Outputs

`modelInst` Integer: model instance identifier.

See Also

`miSaveAscii`, `minst`, `miLoad`.

Example

```
mi = miLoadAscii("miFile.txt");
```

3.4.12 The miSave Command

Description

Saves a model instance in a binary file.

Usage

```
miSave(modelInst, filename);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>filename</code>	String: name of binary file in which to store the model instance

Outputs

None.

See Also

`miLoad`, `minst`, `miSaveAscii`.

Example

```
miSave(mi, "miFile");
```

Details

If the file already exists, it is written over.

3.4.13 The `miSaveAscii` Command

Description

Saves a model instance in a text file.

Usage

```
miSaveAscii(modelInst, filename);
```

Inputs

`modelInst` Integer: model instance identifier.

`filename` String: name of text file in which to store the model instance

Outputs

None.

See Also

`miLoadAscii`, `minst`, `miSave`.

Example

```
miSaveAscii(mi, "miFile.txt");
```

Details

If the file already exists, it is written over. The ascii version of a model instance is platform independent and human readable, but long and inefficient.

3.4.14 The `minst` Command

Description

Creates an instance of a particular model specification.

Usage

```
modelInst = minst(modelSpecName, unknownNames, knowns);
```

Inputs

`modelSpecName` String: name of model specification

`unknownNames` List of strings: user provided names for unknown quantities

`knowns` List of matrices: user provided matrices of known quantities

Outputs

`modelInst` Integer: model instance identifier.

Example

```
a = 1 1 2;  
s = 1 1,2 3,3 3;  
myMI = minst('iidfs', 'pi', a, s);
```

Details

The available model specifications are described in Chapter 2. For each model specification, there is a section “Creating a Model Instance” with the relevant information, namely

- The name of the model specification.
- The order in which the user specifies the names for the unknown quantities of the model.
- The order in which the user provides the matrices giving the values of the known quantities of the model.

3.4.15 The `mlike` Command

Description

Computes various estimates of the marginal likelihood for a model instance, with numerical standard errors.

Usage

```
ml, mlse = mlike(modelInst);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>p</code>	Vector of length L : truncation parameters (optional)
<code>taper</code>	Vector of length K : taper half-widths (optional)

Outputs

<code>ml</code>	Vector of length L : marginal likelihood estimates
<code>mlse</code>	L by $K + 1$ matrix: numerical standard error estimates

See Also

`postsim`, `postsimHM`.

Example

```
/* Use default truncation and taper values */
ml, mlNSE, mlHM, mlNSEHM = mlike(mi);
/* Use alternate truncation values */
p = 0.1 0.3 0.5 0.7 0.9;
ml, mlNSE, mlHM, mlNSEHM = mlike(mi, p);
/* Use alternate truncation and taper values */
taper = 4.0 8.0;
ml, mlNSE, mlHM, mlNSEHM = mlike(mi, p, taper);
```

Details

The method used is a modification described in Geweke [5] of the method proposed in Gelfand and Dey [2].

The truncation parameters $p_l \in [0, 1]$ index the truncated multivariate normal distribution $f(\cdot)$ discussed in Geweke [5]. For each p_l , `mlike` generates (internally) an unweighted vector (z_1^l, \dots, z_M^l) , where M is the number of posterior samples in the given model instance.

For each l , `mlike` calculates the sample mean \bar{z}_l and numerical standard errors $(\tilde{\tau}_1^l, \dots, \tilde{\tau}_M^l)$ from (z_1^l, \dots, z_M^l) and $(\lambda_1, \dots, \lambda_K)$ in the same way that `expect1` calculates (τ_0, \dots, τ_K) from (z_1, \dots, z_M) , a vector of equal log weights, and $(\lambda_1, \dots, \lambda_K)$. Then for all l , the estimate of the log marginal likelihood is given by

$$\mu_l = -\log \bar{z}_l$$

and for all l and k , the estimate of the numerical standard error for the log marginal likelihood is given by

$$\tau_{kl} = \frac{\tilde{\tau}_{kl}}{\bar{z}_l}$$

When numerical standard error is small, results are not sensitive to the choice of p . In these cases $L = 1$ and $p_1 = 0.5$ will suffice. However the additional computational burden of increasing L is negligible. If you are concerned about standard errors, it is best to use several values of p_l , for example, $p = (0.1, 0.2, \dots, 0.9)$.

3.4.16 The `paretoSim` Command

Description

Generates a sample from a Pareto distribution.

Usage

```
sample = paretoSim(alpha, beta, n);
```

Inputs

<code>alpha</code>	Real scalar: tail parameter of Pareto distribution
<code>beta</code>	Real scalar: location parameter of Pareto distribution
<code>n</code>	Integer: number of draws to generate

Outputs

<code>sample</code>	n by 1 matrix: sample generated from Pareto distribution
---------------------	--

See Also

`dirichletSim`, `gaussianSim`, `gammaSim`, `wishartSim`.

Example

```
alpha = 1.0  
beta = 4.0  
sample = gammaSim(alpha, beta, 1000);
```

Details

Each of the n draws of the sample is a scalar with a pareto distribution. The result is given as a n by 1 matrix.

See Appendix A for the parametrization of the Pareto distribution.

3.4.17 The `postfilter` Command

Description

Filters out previously generated draws from the posterior simulation matrix of a given model instance.

Usage

```
postfilter(modelInst, filter);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>filter</code>	Vector of integers of length n : indices of existing draws to keep

Outputs

None.

Example

```
filter = seqa(101,1,898)
postfilter(mi, filter);
```

Details

The i th draw of the posterior simulation matrix is kept if and only if $i = f_j$ for some j from 1 to n .

3.4.18 The `postsim` Command

Description

Generates or appends to the posterior simulation matrix of a given model instance.

Usage

```
postsim(modelInst, m, n);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>m</code>	Integer: number of posterior draws to record
<code>n</code>	Integer: number of posterior draws to generate for each one recorded

Outputs

None.

See Also

`minst`, `postfilter`, `mlike`, `priorsim`, `postsimHM`, `extract`.

Example

```
postsim(mi, 1000, 1);
```

Details

Generates draws of unknown quantities from their posterior distribution. Generates mn new posterior draws, and appends every n th draw to the posterior simulation matrix. If there are any draws from a previous invocation of `postsim`, the first new draw comes from the transition kernel of the Markov chain used for posterior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the posterior draws.

3.4.19 The `postsimHM` Command

Description

Generates or appends to the posterior HM simulation matrix of a given model instance.

Usage

```
postsimHM(modelInst, m, n, scalePrecision);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>m</code>	Integer: number of posterior draws to record
<code>n</code>	Integer: number of posterior draws to generate for each one recorded
<code>scalePrecision</code>	Real scalar: factor used to rescale the precision matrix of the random walk innovation

Outputs

None.

See Also

`minst`, `mlike`, `postsim`, `extract`.

Example

```
postsimHM(mi, 1000, 1, 10.0);
```

Details

Generates draws of unknown quantities from their posterior distribution using a Gaussian random walk Metropolis chain with proposal covariance proportional to the sample covariance of draws from the posterior simulation matrix. Generates mn new posterior draws, and appends every n th draw to the posterior simulation matrix. If there are any draws from a previous invocation of `postsimHM`, the first new draw comes from the transition kernel of the Markov chain used for posterior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the posterior HM draws.

3.4.20 The `priorRobust` Command

Description

Calculates upper and lower bounds on the mean of a posterior function of interest, as the prior distribution is varied from its original specification.

Usage

```
mean, std, U, L, Ut, Lt = priorRobust(logWeight, sample, factors);
```

Inputs

<code>logWeight</code>	Vector of length m : log weights
<code>sample</code>	Vector of length m : posterior sample of some scalar function of interest
<code>factors</code>	Vector of length n : bound factors for robustness analysis

Outputs

<code>mean</code>	Real scalar: posterior sample mean for original prior specification
<code>std</code>	Real scalar: posterior sample standard deviation for original prior specification
<code>U</code>	Vector of length n : exact upper bounds
<code>L</code>	Vector of length n : exact lower bounds
<code>Ut</code>	Vector of length n : asymptotic upper bounds
<code>Lt</code>	Vector of length n : asymptotic lower bounds

Example

```
K = 5.0 10.0 20.0;
mean, std, U, L, Ut, Lt = priorRobust(lw, beta, K);
```

Details

For each bound factor, calculates exact lower and upper bounds and asymptotic lower and upper bounds for the posterior mean. For each bound parameter k_i , `priorRobust` calculates exact lower and upper bounds L_i and U_i for the posterior mean of the function of interest g , for the following set of prior density kernels.

$$\left\{ p^*(\cdot) : \frac{1}{k_i} p(\theta) \leq p^*(\theta) \leq k_i p(\theta) \quad \forall \theta \in \Theta \right\}$$

where $p(\cdot)$ is the actual prior density. It uses the algorithm described in Geweke and Petrella [6]. Also for each k_i , `priorRobust` calculates asymptotically valid

lower and upper bounds \tilde{L}_i and \tilde{U}_i , using the results of DeRobertis and Hartigan [1].

3.4.21 The `priorfilter` Command

Description

Filters out previously generated draws from the prior simulation matrix of a given model instance.

Usage

```
priorfilter(modelInst, filter);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>filter</code>	Vector of integers of length n : indices of existing draws to keep

Outputs

None.

Example

```
filter = seqa(101,1,898)
priorfilter(mi, filter);
```

Details

The i th draw of the prior simulation matrix is kept if and only if $i = f_j$ for some j from 1 to n .

3.4.22 The priorsim Command

Description

Generates or appends to the prior simulation matrix of a given model instance.

Usage

```
priorsim(modelInst, m, n);
```

Inputs

<code>modelInst</code>	Integer: model instance identifier.
<code>m</code>	Integer: number of prior draws to record
<code>n</code>	Integer: number of prior draws to generate for each one recorded

Outputs

None.

See Also

`minst`, `priorfilter`, `postsim`, `extract`.

Example

```
priorsim(mi, 1000, 1);
```

Details

Generates draws of unknown quantities from their prior distribution. Generates mn new prior draws, and appends every n th draw to the prior simulation matrix. If there are any draws from a previous invocation of `priorsim`, the first new draw comes from the transition kernel of the Markov chain used for prior simulation. Otherwise, it comes from the initial distribution of the Markov chain.

Use the `extract` command to obtain the prior draws.

3.4.23 The `setseedconstant` Command

Description

Sets the seeds of the random number generators to a constant value.

Usage

```
setseedconstant;
```

Inputs

None.

Outputs

None.

See Also

```
setseedtime.
```

Example

```
setseedconstant;
```

Details

This is useful for ensuring that repeated invocations of a command generating random values lead to the same results.

3.4.24 The `setseedtime` Command

Description

Sets the seeds of the random number generators to the number of seconds since the beginning of 1970.

Usage

```
setseedtime;
```

Inputs

None.

Outputs

None.

See Also

```
setseedconstant.
```

Example

```
setseedtime;
```

Details

This is useful for ensuring that repeated invocations of a command generating random values lead to different results.

3.4.25 The `weightedSmooth` Command

Description

Estimates a univariate density function for a weighted random sample, using a kernel smoothing algorithm adapted to weighted samples.

Usage

```
x, y = weightedSmooth(logWeight, sample);
```

Inputs

<code>logWeight</code>	Vector of length M : log weights
<code>sample</code>	Vector of length M : a posterior sample of some function of interest
<code>ktype</code>	String: kernel type (optional)
<code>krange</code>	String: kernel range type (optional)
<code>wf</code>	Real scalar: window width fraction (optional)
<code>nplot</code>	Integer: number of ordered pairs to generate (optional)
<code>range_a1</code>	Real scalar: left bound range parameter (optional)
<code>range_a2</code>	Real scalar: right bound range parameter (optional)

Outputs

<code>x</code>	Vector of length N : ordinate values
<code>y</code>	Vector of length N : abscissa values

Example

```
x, y = weightedSmooth(lw, z);
nplot = 2000
ktype = triangular
x, y = weightedSmooth(lw, z);
```

Details

The estimated density at a point z is

$$f(z) = \frac{\sum_{m=1}^M w_m K\left(\frac{z-z_m}{h}\right)}{h \sum_{m=1}^M w_m}$$

The functional form of the kernel function K depends on the value of `ktype` according to Table 3.1.

Table 3.1: Values of `Ktype`

<code>Ktype</code>	K
<code>uniform</code>	$K(t) = \frac{1}{2}\chi_{(-1,1)}(t)$
<code>triangle</code>	$K(t) = (1 - t)\chi_{(-1,1)}(t)$
<code>biweight</code>	$K(t) = \frac{15}{16}(1 - z^2)^2\chi_{(-1,1)}(t)$

For any set S , the function $\chi_S(\cdot)$ is a set membership indicator function.

The value h is given by

$$h = \lambda(q_{\frac{3}{4}} - q_{\frac{1}{4}})$$

where q_α denotes the α 'th sample quantile of z .

The `weightedSmooth` command generates N ordered pairs (x_i, y_i) . The values x_i are evenly spaced between x_{min} and x_{max} , determined by `Krange` according to Table 3.2. The values y_i satisfy $y_i = f(x_i)$.

Table 3.2: Values of `Krange`

<code>Krange</code>	x_{min}	x_{max}
<code>quantile</code>	q_{a_1}	q_{a_2}
<code>absolute</code>	a_1	a_2

For most plotting routines, N should be in the range of 200 to 400. The choice of λ depends on how smooth the resulting plot is desired to be. As with all kernel smoothing methods, some experimentation will probably be necessary. The greater the number of simulations available, the smaller λ can be and still retain visual smoothness. It is generally easier to use the `Krange=quantile` option and specify a_1 in the range .001 to .01 and a_2 in the range .99 to .999; this will include the important part of the estimated density while not wasting space on the plot for points where the density is small.

3.4.26 The `wishartSim` Command

Description

Generates a sample from a Wishart distribution.

Usage

```
sample = wishartSim(A, nu, n);
```

Inputs

<code>A</code>	m by m matrix: inverse scale parameter of Wishart distribution
<code>nu</code>	Real scalar: degrees of freedom parameter of Wishart distribution
<code>n</code>	Integer: number of draws to generate

Outputs

<code>sample</code>	n by m^2 matrix: sample generated from Wishart distribution
---------------------	---

See Also

`paretoSim`, `gaussianSim`, `gammaSim`, `dirichletSim`.

Example

```
A = 1.0 0.0,0.0 1.0;  
nu = 100  
sample = wishartSim(A, nu, 1000);
```

Details

Each of the n draws of the sample is an m by m matrix with a Wishart distribution. The result is given as a m by m^2 matrix.

See Appendix A for the parametrization of the Wishart distribution.

Chapter 4

A BACC Tutorial

In order to answer commonly asked questions, this chapter contains a step-by-step tutorial with explanations of what each step is doing and what each term means.

4.1 Linking to BACC

To use BACC within Gauss, you must first give Gauss access to the BACC library by typing

```
library libPCBACC;
```

and then set-up the BACC dynamically linked library by issuing the command

```
initPCBACC;
```

4.2 Loading Known Quantities

To begin using BACC you need to specify the known quantities. These quantities can be entered directly into Gauss, loaded from a Gauss file, or loaded from a text file. The next lines load the prior matrices and data matrices for the normal linear model. The files for this example are in the `test` directory contained in the BACC zipfile for Gauss users.

```
load betahd []="betahd.mtx";
load Hhd []="Hhd.mtx";
Hhd=reshape(Hhd,5,5);
load nuhd []="nuhd.mtx";
load shd []="shd.mtx";
load Xhd []="Xhd.mtx";
Xhd=reshape(Xhd,39,5);
load yhd []="yhd.mtx";
```

4.3 Creating a Model Instance

Next, a “model instance” is created. A model instance can be thought of as a box containing everything that BACC needs to know about a particular model. This includes the type of the model, i.e., normal linear model, a list of names for the knowns, and the names of the known quantities. The following lines specify a name for the model (`mysim`), names for the unknowns (contained in `names`), names of the knowns (contained in `givens`), and then create an instance of the normal linear model (`nlm`).

```
model="mysim";
names={"beta" "h"};
givens={"betahd" "Hhd" "nuhd" "shd" "Xhd" "yhd"};
minst(model,"nlm",names,givens);
```

4.4 Simulating the Model

Simulation can be performed from the prior or the posterior of a model using the commands `priorsim` and `postsim`. After posterior simulation has been done, it is also possible to sim-

ulate the posterior using a random walk Hastings-Metropolis algorithm. This is done using the command `postsimhm`. If you wish to compare output across runs, it is a good idea to set the seeds of the random number generators. This is done using the `setSeedConstant` command. Strictly speaking, this sets the seeds of another routine that then sets the seeds of the random number generators.

First, generate 5000 samples from the prior.

```
setSeedConstant;
priorsim(model,5000,1);
```

Now generate 1000 samples from the posterior.

```
setSeedConstant;
postsim(model,1000,1);
```

To filter out posterior samples you can specify the numbers of the samples you wish to keep. For example, to drop the first 100 samples you would create a vector containing the numbers 101 to 1000.

```
filt=seqa(101,1,900);
```

Then pass this list of samples you wish to keep to the `postfilter` routine.

```
postfilter(model,filt);
```

Now you are left with 900 posterior samples. Add another 4100 samples.

```
setSeedConstant;
postsim(model,4100,1);
```

We can also simulate 10000 samples from the posterior using a random walk Hastings-Metropolis algorithm.

```
setSeedConstant;
postsimhm(model,10000,1,1);
```

4.5 Computing Marginal likelihoods

The `mlike` command can report several log marginal likelihood values for a model instance. First, if the model has an analytic marginal likelihood, then the exact marginal likelihood is printed. Then, for either a default truncation parameter or for a user specified vector of truncation parameters, `mlike` computes an estimate of the log marginal likelihood for each truncation parameter. Finally, if there are `postsimhm` samples as well, then `mlike` also reports estimates of the log marginal likelihood based on those samples. To use the default

truncation parameter and compute the log marginal likelihoods type

```
p={};
{ml,mlnse,mlhm,mlnsehm}=mlike(mi,p);
```

or to specify a vector of truncation parameters type (for example)

```
p={0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9};
{ml,mlnse,mlhm,mlnsehm}=mlike(mi,p);
```

4.6 Computing Moments of Functions of Interest

To compute the posterior mean of the first element of `beta` you must first extract the vector of logweight values and the appropriate beta elements. This is done as follows:

```
lw=getLogWeight(model);
beta1=getUnknownVector(model,"beta",1);
```

Then to compute the posterior mean of the first element of `beta`, type

```
{betamean,betasd,nse,rne}=expect1(lw,beta1);
```

To specify taper parameters rather than the use the default values, change the values of the global TAPER variable, for example,

```
TAPER={4.0 8.0};
```

prior to running `expect1`,

More generally, `expect1` can compute the mean and standard deviation of any function of interest. The first argument is a vector of log weight values and the second argument is a vector of function of interest values. In some cases it may be sufficient to simply change the weight vector. For example, to find the mean of `h` conditional on `h` being greater than 0.8, first extract `h`,

```
h = getUnknownScalar(model,"h");
```

and then define the log weight vector as

```
lw = log( mysim.h .> .8 );
```

If `h` is greater than 0.8, then `lw` equals $\log(1) = 0$ for that sample. If `h` is less than or equal to 0.8, then `lw` equals $\log(0)=-\text{Infinity}$ for that sample. Hence, the mean of the prior samples of `h` conditional on `h` being greater than 0.8 is found by

```
{betamean,betasd,nse,rne}=expect1(lw,h);
```

In other cases the second argument might be changed. For example,

```
foi = 1./h;
{betamean,betasd,nse,rne}=expect1(lw,foi);
```

To compute combined sample means and test for the equality of the individual population means use the `expectN` command. For example, to compare the posterior and prior means for the first element of the `beta` vector, first extract the prior simulation values,

```
betaprior=getUnknownVector(model,"betaprior",1);
```

and then run `expectN`,

```
expectN(lw,beta1,lw,betaprior);
```

Note that the same logweight vector was used for both the posterior and prior values as both were zero vectors of the same length. If you had different numbers of prior and posterior simulations you would need different vectors.

4.7 Sensitivity to the Prior

To calculate upper and lower bounds on the mean of a posterior function of interest, as the prior distribution is varied from its original specification type

```
{betam,betasd,exactupper,exactlower,drhupper,drhlower}=priorRobust(lw,beta1);
```

To specify values of the `bounds` parameter other than the default values, set the global variable `KPAR` to different values, for example,

```
KPAR={2 10};
```

before running `priorRobust`.

4.8 Kernel Smoothing of Simulations

To estimate a univariate density function for a weighted random sample, using kernel smoothing, type

```
{x,y}=weightedSmooth(lw,beta1);
```

To view the kernel, load the Gauss graphics library,

```
library pgraph;
```

and then plot the kernel using the `xy` function,

```
xy(x,y);
```

The `weightedSmooth` command has several global variables which can be set to, for example, generate more samples and use a different kernel type. For more details see the documentation for `weightedSmooth`.

4.9 Plotting

Besides plotting the output from `weightedSmooth`, the user can use all of the standard Gauss plotting tools to

- plot histograms of parameters,
- plot “time-series” of loglikelihood values or logprior evaluations,
- and to plot scatterplots of two parameters.

4.10 Saving and Loading Model Instances

To save a model instance called `mysim` to an ascii file called `mysim.txt` use the command

```
miSaveAscii("mysim","mysim.txt");
```

To load a model instance stored in an ascii file called `mysim.txt` and to give it the name `mynewsim`, use the command

```
miLoadAscii("mynewsim","mysim.txt");
```

4.11 Drawing from Various Distributions

The following code demonstrates how to draw from various distributions. See Appendix A for the parameterization of these distributions.

4.11.1 Dirichlet

```
a={1 2 3, 4 5 6};
setSeedConstant;
sample=dirichletSim(a,1000);
print "dirichlet mean";
print sumc(sample)/1000;
```

4.11.2 Gamma

```
setSeedConstant;
sample=gammaSim(3,5,1000);
print "gamma mean";
print sumc(sample)/1000;
```

4.11.3 Gaussian (Multivariate Normal)

```
mean={1,2};
precision = {1 0, 0 1};
setSeedConstant;
sample=gaussianSim(mean,precision,1000);
print "gaussian mean";
print sumc(sample)/1000;
```

4.11.4 Pareto

```
setSeedConstant;
sample=paretoSim(3,5,1000);
print "pareto mean";
print sumc(sample)/1000'';
```

4.11.5 Wishart (Multivariate Chi-squared)

```
scale=precision;
setSeedConstant;
sample=wishartSim(scale,10,1000);
print "wishart mean";
print sumc(sample)/1000;
```


Appendix A

Distributions

This appendix gives the density and mass functions for the distributions used in this document.

A.1 The Dirichlet Distribution

A random vector π of length n has the Dirichlet distribution with parameter vector $\alpha \in \mathfrak{R}_+^n$, denoted $\pi \sim \text{Di}(\alpha)$, if its probability density function is

$$p(\pi|\alpha) = \begin{cases} m^{-1/2} \frac{\Gamma(\sum_{i=1}^m \alpha_i)}{\prod_{i=1}^m \Gamma(\alpha_i)} \prod_{i=1}^m \pi_i^{\alpha_i-1} & \pi \in \Delta_n \equiv \{p \in \mathfrak{R}_+^n : \sum_{i=1}^n p_i = 1\} \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \text{E}[\pi_i|\alpha] &= \frac{\alpha_i}{\sum_{j=1}^n \alpha_j} \\ \text{Var}[\pi_i|\alpha] &= \frac{\text{E}[\pi_i|\alpha](1 - \text{E}[\pi_i|\alpha])}{1 + \sum_{j=1}^n \alpha_j} \\ \text{Cov}[\pi_i, \pi_j|\alpha] &= \frac{-\text{E}[\pi_i|\alpha]\text{E}[\pi_j|\alpha]}{1 + \sum_{k=1}^n \alpha_k} \end{aligned}$$

A.2 The Gamma Distribution

A random scalar λ has the Gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, denoted $\lambda \sim \text{Ga}(\underline{\alpha}, \underline{\beta})$, if its probability density function is

$$p(\lambda|\alpha, \beta) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} & \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[\lambda|\alpha, \beta] &= \frac{\alpha}{\beta} \\ \text{Var}[\lambda|\alpha, \beta] &= \frac{\alpha}{\beta^2} \end{aligned}$$

A.3 The Normal Distribution

A random vector x has the Normal Distribution with mean parameter vector $\mu \in \mathfrak{R}^k$ and positive definite $k \times k$ variance parameter matrix Σ , denoted $x \sim \mathcal{N}(\mu, \Sigma)$, if its probability density function is

$$p(x|\mu, \Sigma) = |\Sigma|^{-\frac{1}{2}} (2\pi)^{-\frac{k}{2}} \exp \left[-\frac{1}{2} (x - \mu)' \Sigma^{-1} (x - \mu) \right] \quad \forall x \in \mathfrak{R}^n$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[x|\mu, \Sigma] &= \mu \\ \text{Var}[x|\mu, \Sigma] &= \Sigma \end{aligned}$$

A.4 The Pareto Distribution

A random scalar x has the Pareto Distribution with parameters $\alpha > 0$ and $\beta \geq 0$, denoted $\theta \sim \text{Pa}(\alpha, \beta)$, if its probability density function is

$$p(\theta|\alpha, \beta) = \begin{cases} \alpha\beta^\alpha \theta^{-(\alpha+1)} & \theta \geq \beta \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[\theta|\alpha, \beta] &= \frac{\alpha\beta}{\alpha - 1} \\ \text{Var}[\theta|\alpha, \beta] &= \frac{\alpha\beta^2}{(\alpha - 1)^2(\alpha - 2)} \end{aligned}$$

A.5 The Poisson Distribution

A discrete random variable x has the Poisson distribution with mean parameter $\lambda > 0$, denoted $x \sim \text{Po}(\lambda)$, if its probability mass function is

$$p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!} & x \in \{0, 1, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given by

$$\begin{aligned} \mathbb{E}[x|\lambda] &= \lambda \\ \text{Var}[x|\lambda] &= \lambda \end{aligned}$$

A.6 The Wishart Distribution

An $m \times m$ random matrix H has the Wishart distribution with positive definite $m \times m$ scale parameter matrix A and degrees of freedom parameter $\nu > m$, denoted $H \sim \text{Wi}(A, \nu)$, if its probability density function is

$$p(H|A, \nu) = \begin{cases} \frac{\pi^{-m(m-1)/4} |A|^{-\nu/2}}{2^{m\nu/2} \prod_{i=1}^m \Gamma(\frac{\nu-i+1}{2})} \cdot |H|^{(\nu-m-1)/2} \exp\left[-\frac{1}{2}\text{tr}(A^{-1}H)\right] & H \text{ p.d.} \\ 0 & \text{otherwise} \end{cases}$$

The mean, and mean of the matrix H^{-1} are given by

$$\begin{aligned} \mathbb{E}[H|A, \nu] &= \nu A \\ \mathbb{E}[H^{-1}|A, \nu] &= \frac{1}{\nu - m - 1} A^{-1} \end{aligned}$$

Appendix B

A Brief Gauss Tutorial

This section gives a brief overview of Gauss, and especially its matrix operations. It is not intended to be a complete tutorial. Consult the Gauss manual for further information.

B.1 Manipulating Matrices

This section gives examples of Gauss commands for manipulating matrices.

1. Creating a matrix:

```
x={1 2 3, 4 5 6};  
print x;  
1.0000000 2.0000000 3.0000000  
4.0000000 5.0000000 6.0000000
```

2. Determining dimensions:

```
print rows(x);  
2.0000000  
print cols(x);  
3.0000000
```

3. Creating special matrices:

```
y=zeros(2,3);  
print y;  
0.0000000 0.0000000 0.0000000  
0.0000000 0.0000000 0.0000000  
z=ones(2,1);  
print z;
```

```

1.0000000
1.0000000
id2=eye(2); /* 2 dimensional identity matrix */
print id2;
1.0000000 0.0000000
0.0000000 1.0000000

```

4. Concatenating matrices:

```

print x|y; /* vertical concatenation */
1.0000000 2.0000000 3.0000000
4.0000000 5.0000000 6.0000000
0.0000000 0.0000000 0.0000000
0.0000000 0.0000000 0.0000000

print x~z; /* horizontal concatenation */
1.0000000 2.0000000 3.0000000 1.0000000
4.0000000 5.0000000 6.0000000 1.0000000

```

5. Transposing matrices:

```

print x';
1.0000000 4.0000000
2.0000000 5.0000000
3.0000000 6.0000000

```

6. Reshaping matrices:

```

print reshape(x,6,1);
1.0000000
2.0000000
3.0000000
4.0000000
5.0000000
6.0000000

print reshape(x,3,2);
1.0000000 2.0000000
3.0000000 4.0000000
5.0000000 6.0000000

```

7. Finding column maxima and minima:

```

print maxc(x);
4.0000000
5.0000000
6.0000000

```

```
print minc(x);
    1.0000000
    2.0000000
    3.0000000
```

8. Indexing elements of matrices:

```
print x[.,1];
    1.0000000
    4.0000000
print x[2,.];
    4.0000000 5.0000000 6.0000000
print x[1,2:3];
    2.0000000 3.0000000
```

9. Creating arithmetic and geometric sequences:

```
print seqa(2,0.5,4); /* arithmetic sequence */
    2.0000000
    2.5000000
    3.0000000
    3.5000000
print seqm(2,0.5,4); /* geometric sequence */
    2.0000000
    1.0000000
    0.5000000
    0.2500000
```

B.2 Other Useful Information

1. Strings must be enclosed in double quotes:

```
s="this is a string";
```

2. The `load` command can be used to read data from ascii files. Suppose the file `data` contains

```
1 3 5 7 9 11
```

Then typing

```
load a[]="data";
print a;
```

will result in

```
1.0000000  
3.0000000  
5.0000000  
7.0000000  
9.0000000  
11.0000000
```

3. Global variables can be deleted in the command mode only. To delete a global variable `x` type

```
delete x;
```

This will cause `x` to be removed from the symbol table.

Bibliography

- [1] DeRobertis, L., and J. A. Hartigan, 1981, "Bayesian Inference Using Intervals of Measures," *The Annals of Statistics* 9: 235-244.
- [2] Gelfand, A.E. and D.K. Dey, 1994, "Bayesian Model Choice: Asymptotics and Exact Calculations," *Journal of the Royal Statistical Society Series B* **56**: 501-514.
- [3] Geweke, J., 1989, "Bayesian Inference in Econometric Models Using Monte Carlo Integration" *Econometrica*, 57, 1317-1340.
- [4] Geweke, J., 1992, "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments," in J.O. Berger, J.M. Bernardo, A.P. Dawid, and A.F.M. Smith (eds.), *Proceedings of the Fourth Valencia International Meeting on Bayesian Statistics*, 169-194. Oxford: Oxford University Press.
- [5] Geweke, J. 1999, "Simulation-Based Bayesian Inference for Economic Time Series," in R.S. Mariano, T. Schuermann and M. Weeks (eds.), *Simulation-Based Inference in Econometrics: Methods and Applications*. Cambridge: Cambridge University Press, forthcoming.
- [6] Geweke, J. and L. Petrella, 1999, "Prior Density Ratio Class Robustness in Econometrics," *Journal of Business and Economic Statistics*, forthcoming.
- [7] Raftery, A.E., 1995, "Hypothesis testing and model selection via posterior simulation," University of Washington working paper.